Knd Chary

UNIT-III

MAGNETO STATICS - I

Biot-Savart's Law and its applications: Static magnetic fields - Biot-Savart's Law - Magnetic field intensity (MFI) - MFI due to a straight current carrying filament - MFI due to circular, square and solenoid current carrying wire - Relation between magnetic flux, magnetic flux density and MFI - Maxwell's equation

Ampere's circuital law and its applications: MFI due to an infinite sheet of current and a long current carrying filament - point form of Ampere's circuital law - Maxwell's equation. Field due to a circular loop, rectangular and square loops.

Introduction:

→ Uptill now static electric fields are discussed. The electrostatic field exists due to the static charges i.e., charges are at rest.

→ The magnetic field exists due to permanenent magnet, which is a natural magnet.
But there is ~~no~~ link absent b/w electric field and magnetic field due to natural magnet.

→ A definite link between electric and magnetic fields was established by Oersted in 1820. As we have noticed, an electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced.

→ A magnetostatic field is produced by a constant current flow (or direct current).

→ Magnetostatics deals with magnetic field produced by Current carrying Conductor.

→ The study of steady magnetic field, existing in a given space produced due to the flow of direct current through a conductor is called "Magnetostatics"

→ A static magnetic field can be produced from a permanent magnet or a current carrying conductor.

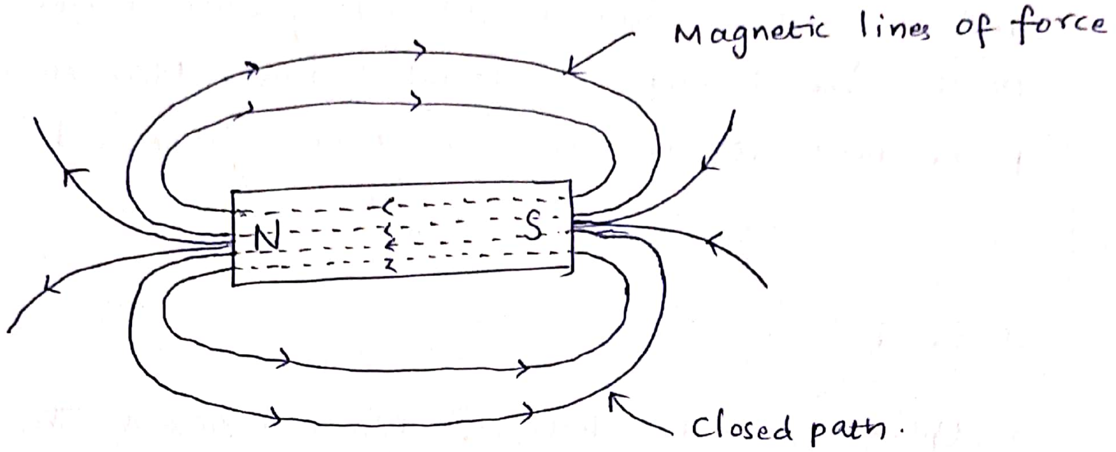


Fig: permanent magnet and magnetic lines of force.

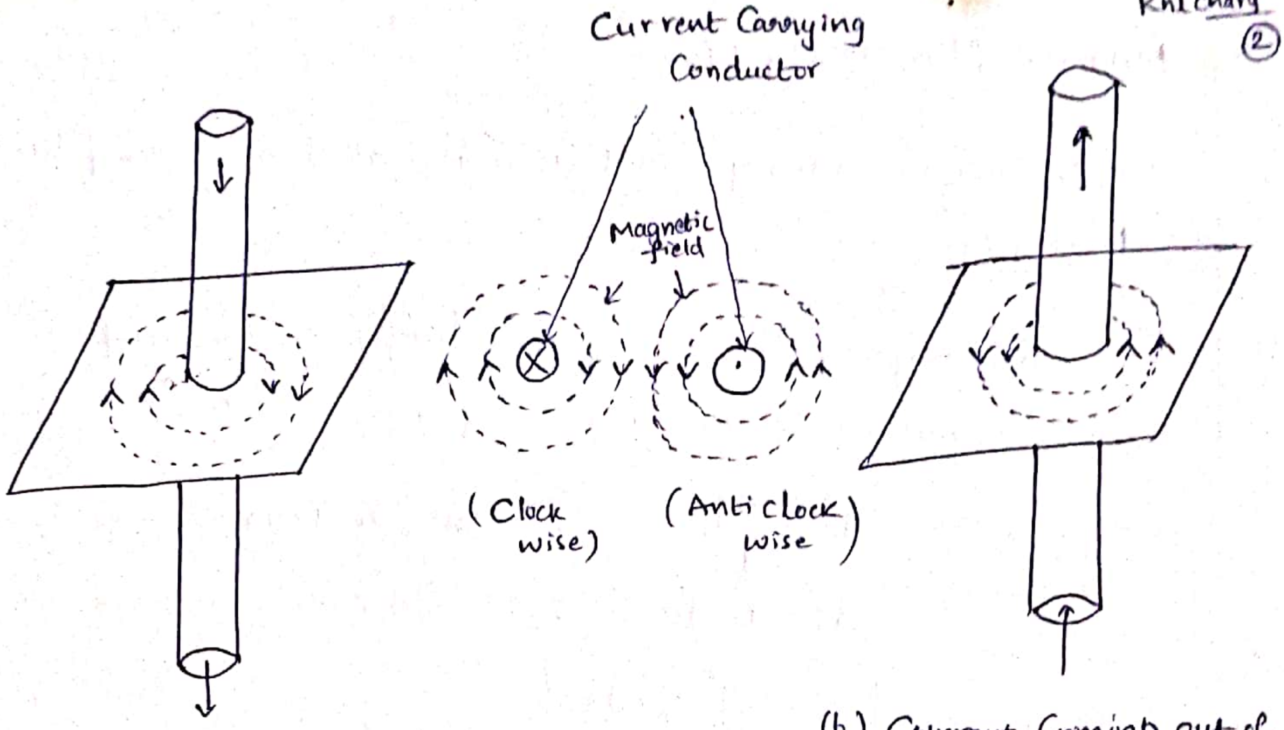
NOTE An isolated magnetic pole can not exist.

→ The magnetic lines of force are also called magnetic lines of flux or magnetic flux lines.

→ Hence every magnetic flux line starting from North pole must end at South pole and Complete the path from South to North internal to the magnet.

NOTE The magnetic flux lines exist in the form of closed loop.

→ Magnetic field: The region around a magnet with in which the influence of the magnet can be experienced is called magnetic field.



(a) Current going into the plane of paper

(b) Current Coming out of the plane of paper.

→ When a straight Conductor Carries a direct current, it produces a magnetic field around it in the form of Concentric circles.

→ ⊗ - Cross indicates that the current direction is going into the plane of the paper away from the Observer.

⊙ - dot indicates that the current direction is Coming out of the plane of the paper Coming towards the observer.

→ Right Hand Thumb Rule is used to determine the direction of field around a current Carrying Conductor.

Thumb → pointing the direction of Current.

Curled fingers → pointing the direction of magnetic lines of flux around the Conductor.

Magnetic flux density ($\bar{B}$):

The magnetic flux per unit area is called magnetic flux density ($\bar{B}$).

→ The magnetic flux through any surface is the surface integral of the normal component of $\bar{B}$.

$$B = \frac{d\phi}{da} \quad Wb/m^2 \text{ or Tesla.}$$

$$d\phi = B \cdot da.$$

$$\boxed{\phi = \iint_S \vec{B} \cdot \vec{da}}$$

Magnetomotive force (M·M·F):

MMF is produced when an electric current flows through a coil of several turns. The M·M·F depends on the current and the number of turns.

$$M{\cdot}M{\cdot}F = NI \quad \text{Ampere Turns.}$$

Magnetic field Intensity ($\bar{H}$) (or) Magnetic field strength

Magnetic field intensity at any point in the magnetic field is defined as the force experienced by the unit north pole of one weber strength, when placed at that point.

(or)

MFI is defined as the ratio of M·M·F to meter length

$$\boxed{\bar{H} = \frac{M{\cdot}M{\cdot}F}{\text{length}} = \frac{NI}{l}} \quad N/Wb \text{ or } AT/m \text{ or } A/m$$

Units: Newton/weber or AmperTurns/meter or Ampere/meter.

Reluctance: Reluctance is the opposition to the establishment of magnetic flux and canbe defined as the ratio of M.M.F to the flux produced.

$$\boxed{\text{Reluctance}, R = \frac{M.M.F}{\text{magnetic flux}} = \frac{NI}{\phi} \quad AT/wb}$$

→ There are two major laws governing magnetostatic fields:
(1) Biot-savart's law and (2) Ampere's Circuit law.

→ Like Coulomb's law, Biot-Savart's law is the general law of magnetostatics.

→ Just as Gauss's law is a special Case of Coulomb's law, Ampere's law is a Special Case of Biot-Savart's law and easily applied in problems involving symmetrical Current distribution.

BIOT - SAVART'S LAW

→ Biot - Savart's law states that the differential magnetic field intensity $d\bar{H}$ produced at a point P, by the differential current element $I dl$ is ~~pro~~

(i) proportional to the product $I dl$ and the sine of the angle α between the element and the line joining P to the element and (ii) inversely proportional to the square of the distance R between P and the element.

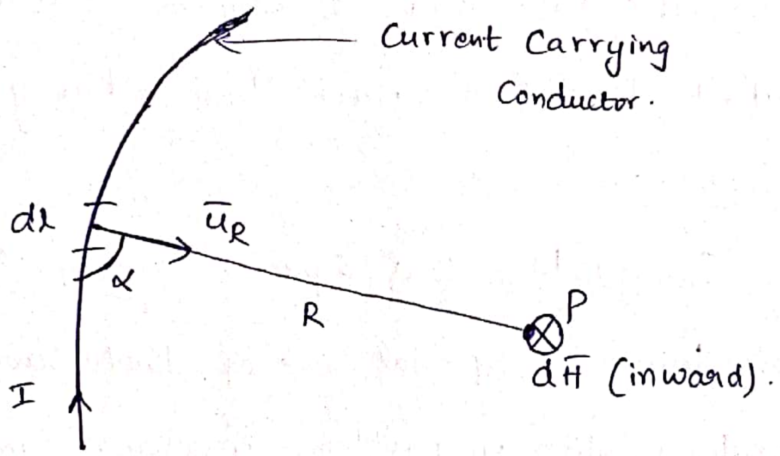


Fig: Magnetic field $d\bar{H}$ at P due to Current element $I dl$.

dl → Differential length

$I dl$ → Differential Current element

$d\bar{H}$ → Differential magnetic field intensity

R → Distance between P and the element.

α → Angle b/w the element and the line joinging P to the element.

u_R → Unit vector directed from element to P.

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Mathematical Representation

$$d\bar{H} \propto \frac{I\,dl \sin\alpha}{R^2}$$

$$d\bar{H} = \frac{K I\,dl \sin\alpha}{R^2}$$

where K = Constant of proportionality

$$= \frac{1}{4\pi}$$

$$d\bar{H} = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

$$d\bar{H} = \frac{I\,dl \times \bar{u}_R}{4\pi R^2}$$

$$d\bar{H} = \frac{I\,dl \times \bar{R}}{4\pi R^3} \qquad \left[\because \bar{u}_R = \frac{\bar{R}}{|R|}\right]$$

$$\Rightarrow \boxed{d\bar{H} = \frac{I\,dl \times \bar{u}_R}{4\pi R^2} = \frac{I\,dl \times \bar{R}}{4\pi R^3}} \quad A/m.$$

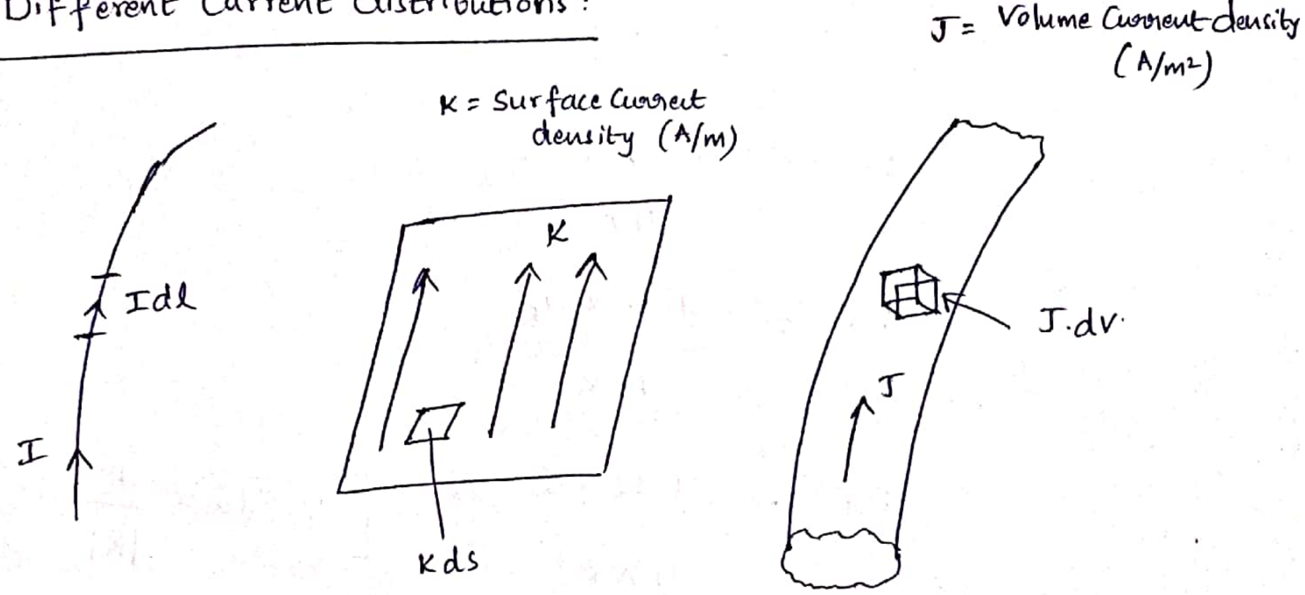
→ The direction of $d\bar{H}$ can be determined by the right-hand rule with the right-hand thumb pointing the direction of the current and right-hand curled fingers encircling the wire in the direction of $d\bar{H}$.

→ It is customary to represent the direction of magnetic field intensity $\bar{H}$ (or Current I) by a Small circle with a dot or cross sign depending on whether $\bar{H}$ ~~or~~ (or I) is out of page, or into it respectively.

$$\bar{H} = \oint \frac{I dl \times \bar{u}_R}{4\pi R^2} = \oint \frac{I dl \times \bar{R}}{4\pi R^3}$$

$$\bar{B} = \oint \frac{\mu I dl \times \bar{u}_R}{4\pi R^2} = \oint \frac{\mu I dl \times \bar{R}}{4\pi R^3}$$

Different Current distributions:



(a) (b) (c)

The Source elements are related as

$$I dl \equiv K \cdot ds \equiv J \cdot dv$$

Thus interms of distributed Current Sources, the Biot-Savart's law

$$H = \int_L \frac{I dl \times \bar{u}_R}{4\pi R^2} \quad \text{(Line Current)}$$

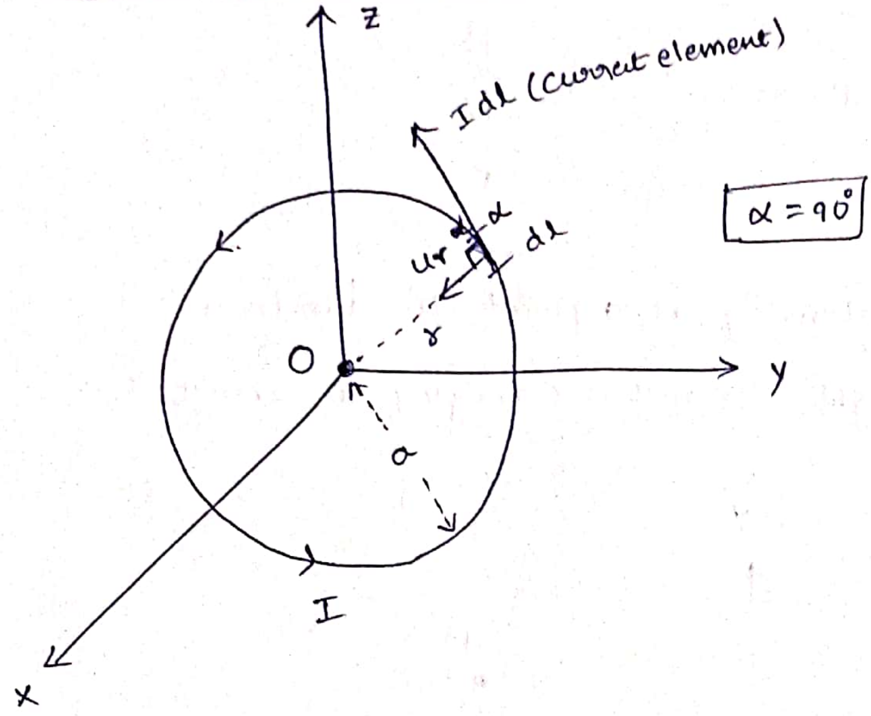
$$H = \int_s \frac{K ds \times \bar{u}_R}{4\pi R^2} \quad \text{(Surface Current)}$$

$$H = \int_v \frac{J dv \times \bar{u}_R}{4\pi R^2} \quad \text{(Volume Current).}$$

where $\bar{u}_R$ is a unit vector pointing from differential element of Current to the point of interest.

Examples :

(1) Field Intensity $\bar{H}$ at the Centre of Circular wire Carrying Current, I :



→ The field intensity at O is given by $\bar{H} = \oint d\bar{H}$

where $d\bar{H}$ is the field intensity at O due to any Current element Idl.

According to Biot-Savart's Law

$$d\bar{H} = \frac{I dl \sin 90°}{4\pi a^2} \hat{K} \qquad \left[\because \begin{array}{l} \alpha = 90° \\ r = a \end{array} \right]$$

$$d\bar{H} = \frac{I dl}{4\pi a^2} \hat{K}$$

$$\bar{H} = \oint d\bar{H} = \oint \frac{I dl}{4\pi a^2} \hat{K}$$

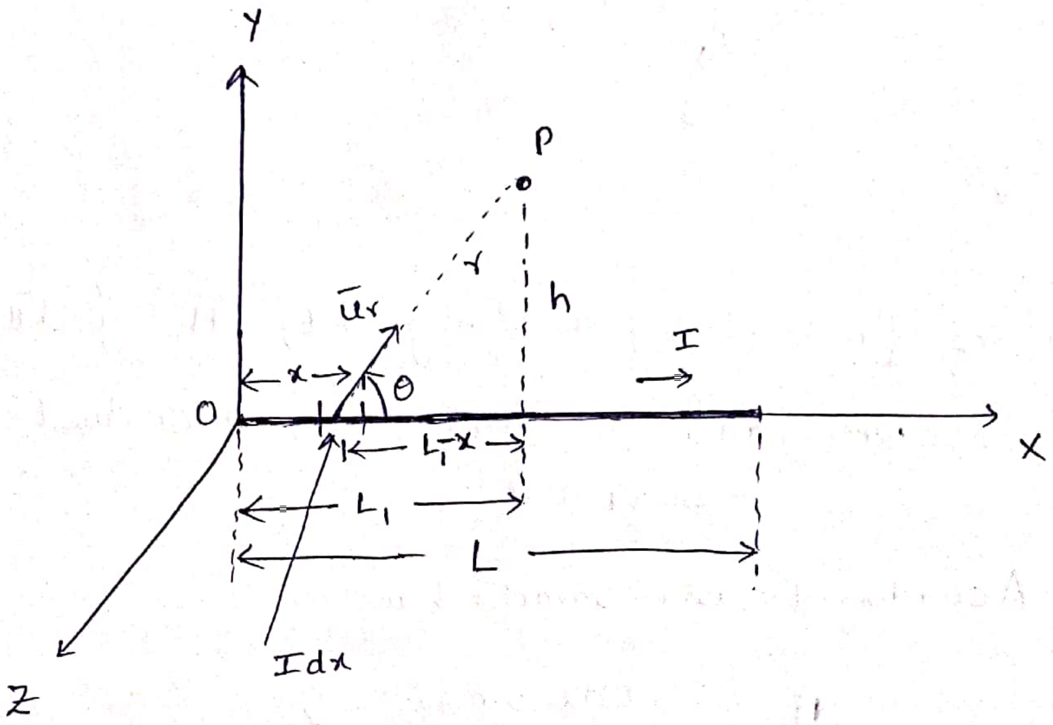
→ Field is directed perpendicular to the XY-plane. i.e., along Z-axis

$$\bar{H} = \hat{k} \cdot \frac{I}{4\pi a^2} \oint dl$$

$$= \hat{k} \cdot \frac{I}{\cancel{4}\pi a^{\cancel{2}}} (\cancel{2}\cancel{\pi} a)$$

$$\boxed{\bar{H} = \frac{I}{2a} \hat{k}}$$

(2) Field Intensity at a point 'p' due to a Straight Conductor Carrying a current, I :-



→ Let 'p' be any point distant 'h' from the straight conductor in XY-plane.

→ Let us take the conductor to be along the x-axis from O; length of the conductor, L.

→ Consider any current element "Idx" at a distance x from O and let its distance to p be 'r'.

According to Biot-savart's law,

$$dH_p = \frac{I\,dx}{4\pi r^2}\,\hat{i} \times \bar{u}_r$$

$$dH_p = \frac{I\,dx \sin\theta}{4\pi r^2}\,\hat{K} \quad \left[\because \hat{i} \times \bar{u}_r = (\sin\theta)\,\hat{K}\right.$$

Total field intensity at 'p' is

$$H_p = \int dH_p$$

$$H_p = \hat{K}\,\frac{I}{4\pi}\int \frac{\sin\theta}{r^2}\,dx$$

From the diagram

$$\tan\theta = \frac{h}{L_1 - x}$$

$$\Rightarrow \cot\theta = \frac{L_1 - x}{h}$$

$$\Rightarrow L_1 - x = h\cot\theta$$

$$-dx = -h\,\mathrm{cosec}^2\theta\,d\theta$$

$$\boxed{dx = h\,\mathrm{cosec}^2\theta\,d\theta}$$

$$\sin\theta = \frac{h}{r}$$

$$\Rightarrow \mathrm{Cosec}\theta = \frac{r}{h}$$

$$\Rightarrow \boxed{r = h\,\mathrm{cosec}\theta}$$

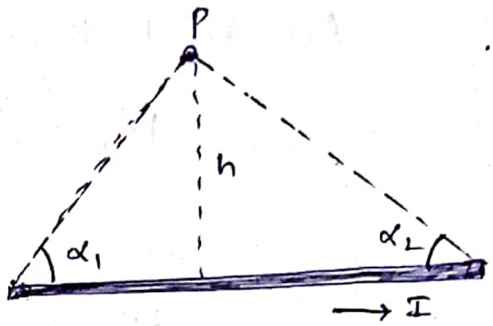
Substitute dx & r in the above equation

$$H_p = \hat{K}\,\frac{I}{4\pi}\int \frac{\sin\theta}{h^{\cancel{2}}\,\cancel{\mathrm{cosec}^2\theta}}\cdot \cancel{h}\,\cancel{\mathrm{cosec}^2\theta}\,d\theta$$

$$= \hat{K}\,\frac{I}{4\pi h}\int \sin\theta \cdot d\theta$$

$$= \hat{K}\,\frac{I}{4\pi h}\left[-\cos\theta\right]_{\theta \text{ for } x=0}^{\theta \text{ for } x=L}$$

$$H_p = \hat{k}\frac{I}{4\pi h}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$



$$H_p = \hat{k}\frac{I}{4\pi h}\left[-\cos(\pi-\alpha_2)+\cos\alpha_1\right]$$

$$H_p = \hat{k}\frac{I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]$$

$$\boxed{H_p = \frac{I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]\hat{k}}$$

NOTE: For infinite long straight conductor carrying current I, the field intensity at p is given by

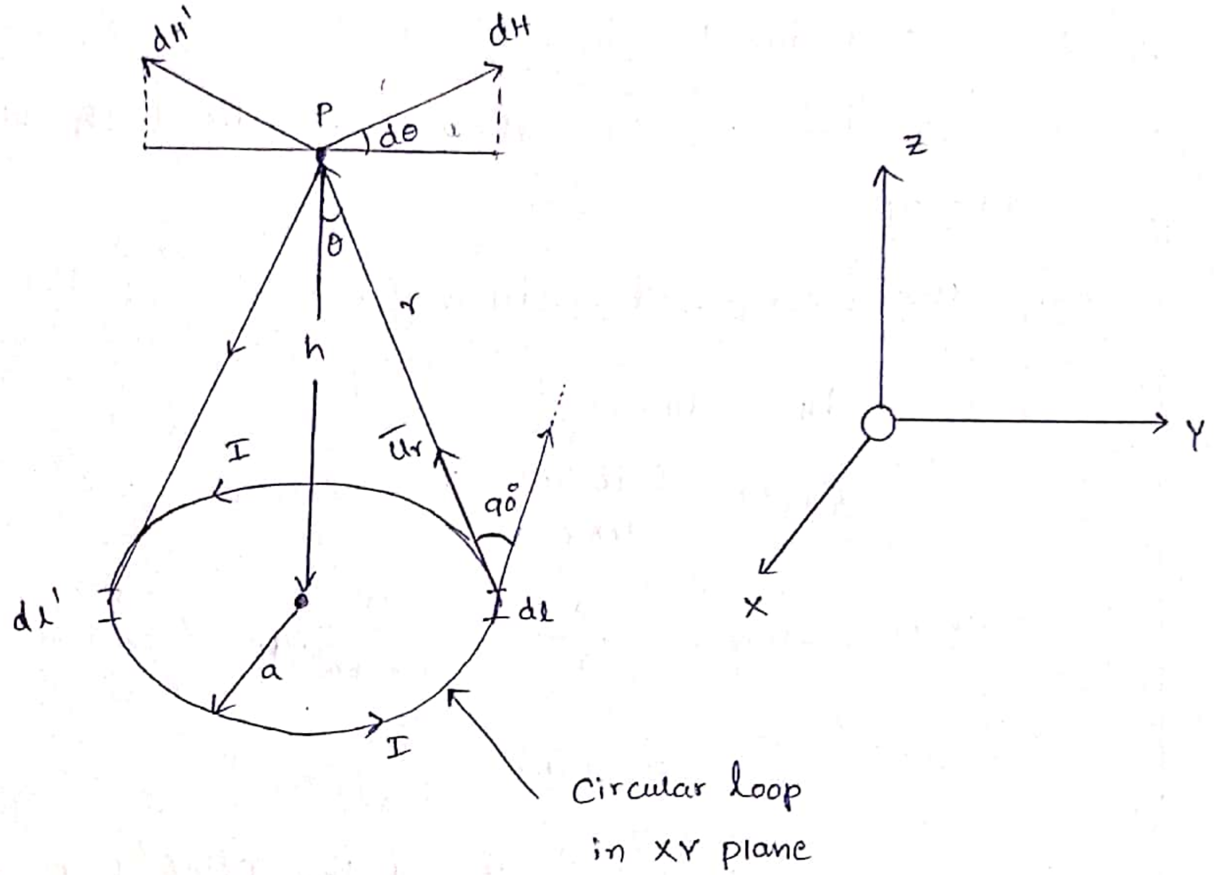
$$\boxed{H_p = \frac{I}{2\pi h}\hat{k}} \qquad [\because \alpha_1=\alpha_2=0]$$

The corresponding flux density expressions are

$$B_p = \frac{\mu_0 I}{4\pi h}\left[\cos\alpha_1+\cos\alpha_2\right]\hat{k}$$

$$B_p = \frac{\mu_0 I}{2\pi h}\hat{k} \quad \text{— for a infinite long conductor.}$$

(3) Field intensity $\bar{H}$ at a point 'p' due to ~~circ~~ Circular loop Carrying Current I :



→ Consider two diametrally opposite elements of the wire loop dl and dl'.

The field intensity at P distant r from the Current element Idl is given by

$$d\bar{H} = \frac{I dl \times \bar{u}_r}{4\pi r^2}$$

As the Vectors $d\bar{l}$ and $\bar{u}_r$ are perpendicular,

$$d\bar{H} = \frac{I dl}{4\pi r^2}$$

→ The field is Oriented at an angle θ to the plane of loop.

→ The diametrally opposite element Idl' will also produce a field of magnitude equal to dH; but its Component parallel to the plane of the loop, though equal is oppositely directed to the field due to element Idl.

→ However, the Components along Z-axis due to the elements add up.

The Z-Component of dH is given by

$$dH_z = dH \sin\theta$$

$$dH_z = \frac{Idl \sin\theta}{4\pi r^2}$$

where $\sin\theta = \frac{a}{r} = \frac{a}{(a^2+h^2)^{1/2}}$

$$r^2 = a^2 + h^2$$

The resultant field-intensity at the point 'p' is given

$$H_p = \oint dH_z$$

$$= \oint \frac{Idl}{4\pi (a^2+h^2)^2} \cdot \frac{a}{(a^2+h^2)^{3/2}}$$

$$= \frac{Ia}{4\pi (a^2+h^2)^{3/2}} \oint dl$$

$$H_p = \frac{Ia}{\cancel{4}^{2}\pi (a^2+h^2)^{3/2}} (2\pi a) \quad \left[\because \oint dl = \text{length of the wire loop}\right]$$

$$\boxed{H_p = \frac{Ia^2}{2(a^2+h^2)^{3/2}} \hat{k}}$$

→ H_p is directed along Z-axis

Note If $h=0$, p Coincides with O, the Centre of the wire loop.

$$\boxed{\bar{H} = \frac{Ia^2}{2a^3}\hat{k} = \frac{I}{2a}\hat{k}}$$

(4) Field $\bar{B}$ at any point along the axis of a Solenoid :

Solenoid:

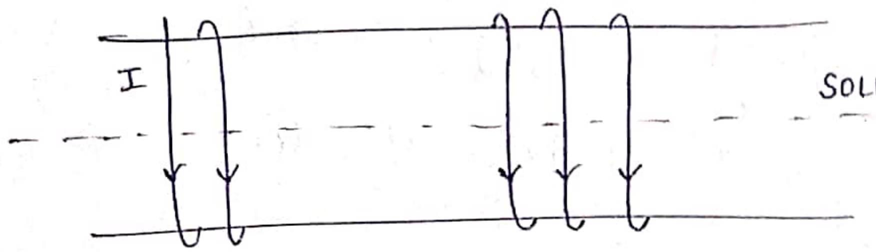
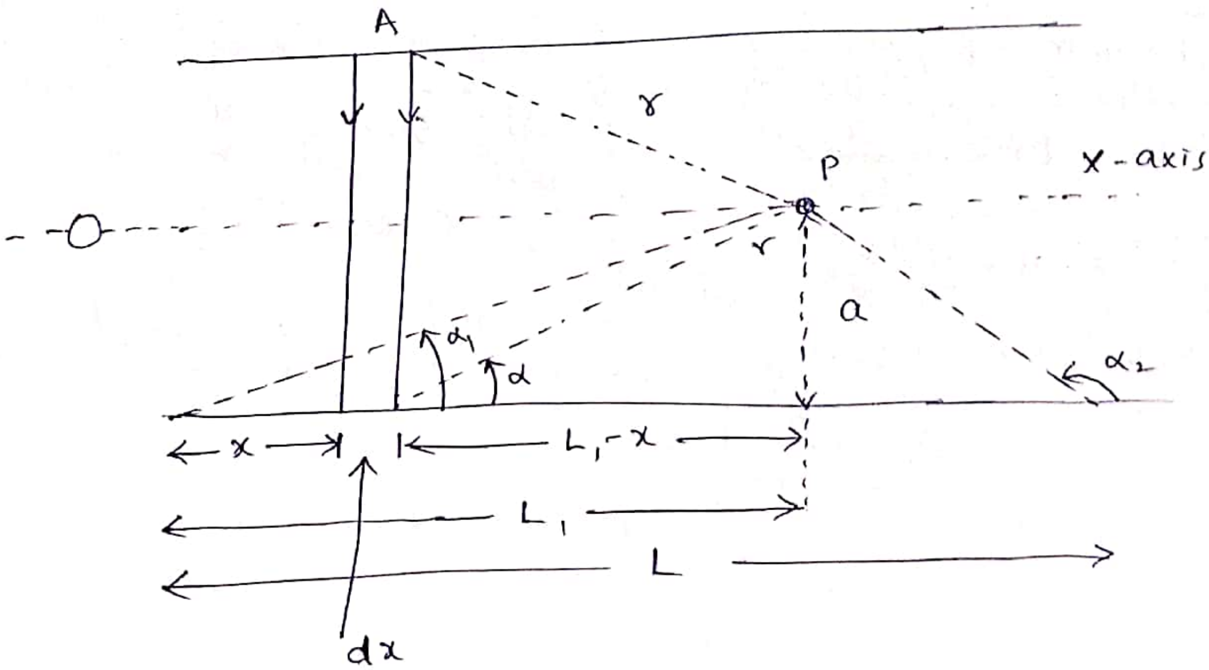


Fig: Solenoid.

" A solenoid is a cylindrically shaped Coil Consisting of a large number of closely spaced turns of insulated wire wound usually on a non-magnetic frame."



→ Let the Solenoid have N turns uniformly distributed over a length L and mean radius of the coil = a.

→ For 'L' length, the no. of turns = N

For unit length, the no. of turns = $\frac{N}{L}$

For 'dx' length, the no. of turns = $\frac{N}{L} dx$

→ As the current is I, we can consider that the current in the coil produces a current sheet with a linear current density

$$\lambda = \frac{NI}{L}.$$

→ The length dx carrying a current

$$I' = \lambda\, dx = \frac{NI}{L} dx.$$

→ Hence, the field $d\bar{B}$ due to the current sheet of length dx is

$$d\bar{B} = \frac{\mu_0 I' a^2}{2r^3}\hat{i}$$

$$d\bar{B} = \hat{i}\,\frac{\mu_0 I a^2}{2r^3}\,\frac{N}{L}\,dx.$$

From the figure

$$\tan\alpha = \frac{a}{L_1 - x}$$

$$\Rightarrow \cot\alpha = \frac{L_1 - x}{a}$$

$$\Rightarrow L_1 - x = a\cot\alpha$$

$$\boxed{dx = a\,\mathrm{cosec}^2\alpha\, d\alpha}$$

$$\sin\alpha = \frac{a}{r}$$

$$\Rightarrow \mathrm{cosec}\,\alpha = \frac{r}{a}$$

$$\Rightarrow \boxed{r = a\,\mathrm{cosec}\,\alpha}$$

$$d\bar{B} = \hat{i}\,\frac{\mu_0 I \cancel{a^2}}{2\cancel{a^2}\,\text{cosec}^{\cancel{3}}\alpha} \cdot \frac{N}{L}\,\cancel{a}\,\text{cosec}^{\cancel{2}}\alpha\, d\alpha$$

$$d\bar{B} = \hat{i}\,\frac{\mu_0 N I}{2L}\,\sin\alpha\, d\alpha$$

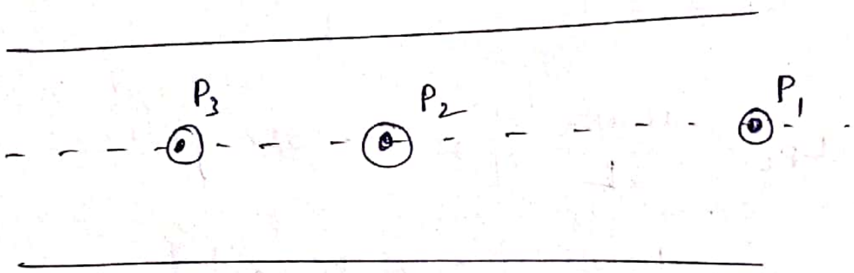
The resultant field at 'p' is

$$\bar{B} = \int_{\alpha_1}^{\alpha_2} d\bar{B} = \int_{\alpha_1}^{\alpha_2} \hat{i}\,\frac{\mu_0 N I}{2L}\,\sin\alpha\, d\alpha$$

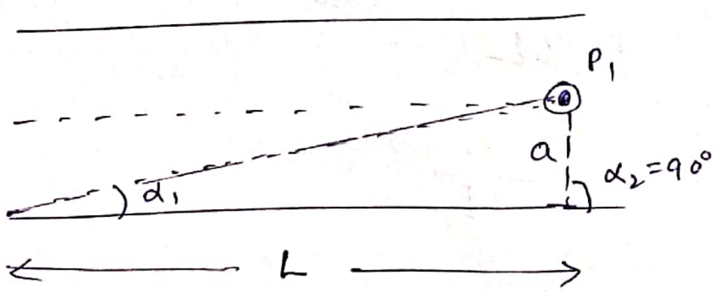
$$\bar{B} = \frac{\mu_0 N I}{2L}\left[-\cos\alpha_1\right]_{\alpha_1}^{\alpha_2}\,\hat{i}$$

$$\boxed{\bar{B} = \frac{\mu_0 N I}{2L}\left[\cos\alpha_1 - \cos\alpha_2\right]\hat{i}}$$

Let us consider three particular locations of P.



Case(i): Let P be at one of the ends of the solenoid.



$$\cos\alpha_1 = \frac{L}{\sqrt{L^2+a^2}}$$

If $L >> a$

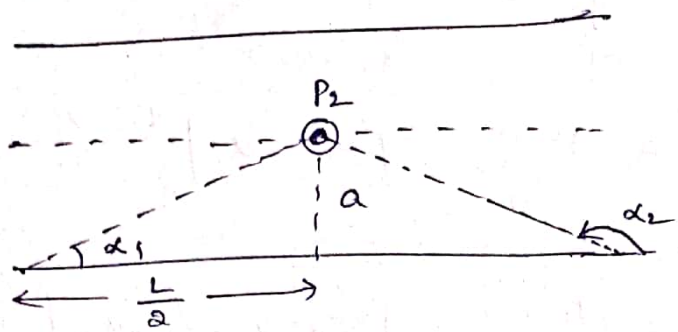
$\cos\alpha_1 = 1$

$\cos\alpha_2 = \cos 90^\circ = 1$

$$B_{P_1} = \frac{\mu_0 N I}{2L}\left[1-0\right]$$

$$\boxed{B_{P_1} = \frac{\mu_0 N I}{2L}\,\hat{i}}$$

<u>Case (2):</u> Let 'p' be at the centre of the solenoid



$$\cos\alpha_1 = \frac{L/2}{\sqrt{\left(\frac{L}{2}\right)^2 + a^2}}$$

$$\cos(\pi-\alpha_2) = \cos\alpha_1$$

$$\Rightarrow \cos\alpha_2 = -\cos\alpha_1$$

If $L/2 >> a$

$$\cos\alpha_1 = 1$$

$$B_{P_2} = \frac{\mu_0 N I}{2L}\left[\cos\alpha_1 - \cos\alpha_2\right]\hat{i}$$

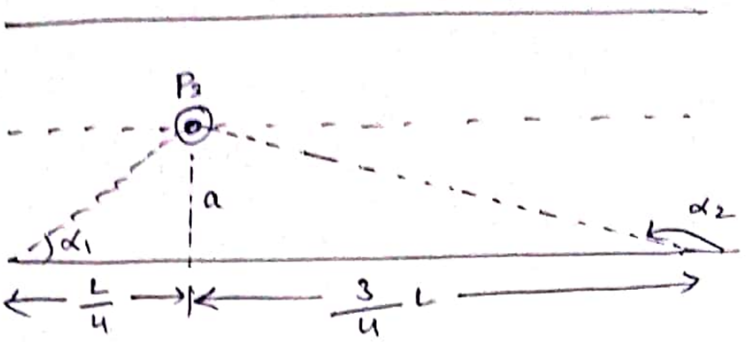
$$= \frac{\mu_0 N I}{2L}\left[\cos\alpha_1 + \cos\alpha_1\right]\hat{i}$$

$$= \frac{\mu_0 N I}{\cancel{2}L}\,\cancel{2}\cos\alpha_1\,\hat{i}$$

$$= \frac{\mu_0 N I}{\cancel{2}L}(1)\,\hat{i}$$

$$\boxed{B_{P_2} = \frac{\mu_0 N I}{L}\,\hat{i}}$$

Case (3): Let 'p' be a point on the axis, midway between one end and centre of the solenoid.



$$\cos\alpha_1 = \frac{L/4}{\sqrt{\left(\frac{L}{4}\right)^2 + a^2}}$$

$$= \frac{L}{\sqrt{L^2 + 16a^2}}$$

$$\cos(\pi - \alpha_2) = \frac{3/4\, L}{\sqrt{\left(\frac{3}{4}L\right)^2 + a^2}}$$

If $\frac{3}{4}L >> a$.

$$\cos(\pi - \alpha_2) = 1$$

$$-\cos\alpha_2 = 1$$

$$\boxed{\cos\alpha_2 = -1}$$

$$B_{P_3} = \frac{\mu_0 N I}{2L}\left[\cos\alpha_1 - \cos\alpha_2\right]$$

$$\boxed{B_{P_3} = \frac{\mu_0 N I}{2L}\left[1 + \frac{L}{\sqrt{L^2 + 16a^2}}\right]} \quad \rightarrow (i)$$

Graph:

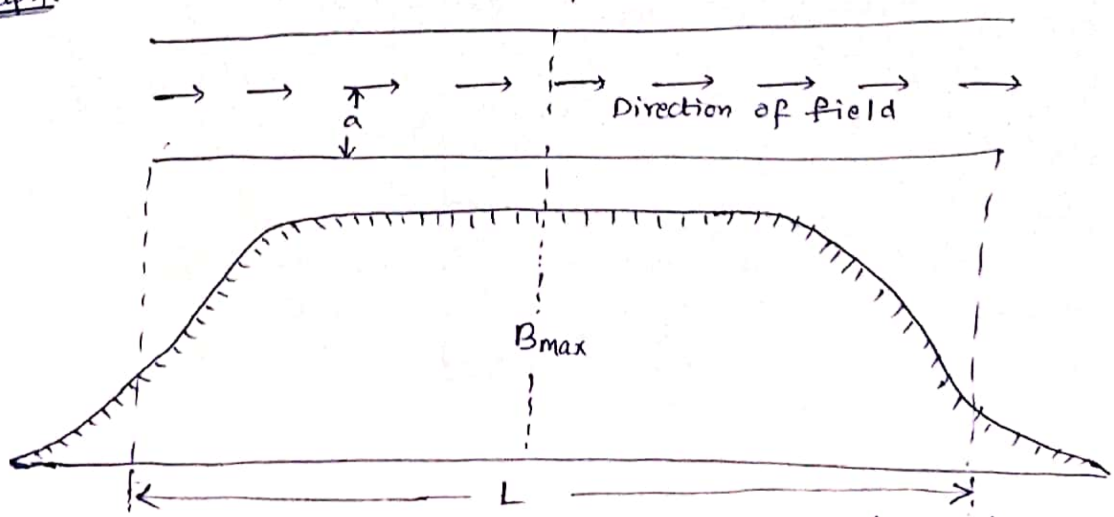


Fig: Variation of B along the axis of a long slender solenoid.

Ampere's Circuital Law (Ampere's Work Law)

→ When the magnetic field has some form of symmetry, the magnetic flux density can be determined with the application of law known as Ampere's Law.

→ In Electrostatics, Complex problems can be solved using a law called Gauss's law.

Gauss's law is useful to obtain $\bar{E}$.

→ Similarly, in Magnetostatics, Complex problems can be solved using a law called Ampere's circuital Law.

Statement:

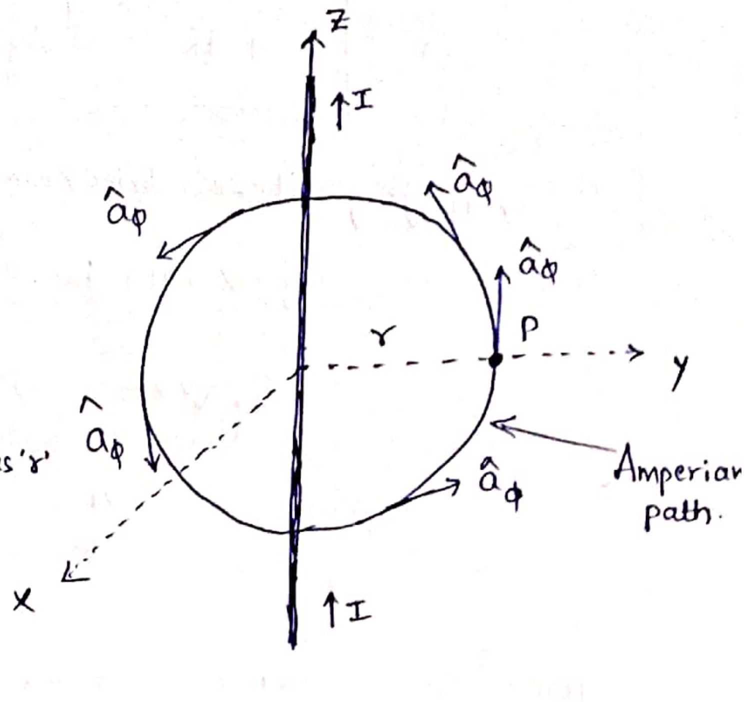
Ampere's circuit law states that the line integral of $\bar{H}$ around a closed path is the same as the net current, I_{enc} enclosed by the path.

$$\boxed{\oint_L H \cdot dl = I_{enc}}$$

proof:

→ Consider a long straight Conductor carrying 'I' placed along z-axis.

→ Consider a closed Circular path or Amperian path of radius 'r' which encloses the straight Conductor carring current, 'I'.



→ Consider $d\bar{l}$ at point 'p' which is in $\hat{a}_\phi$ direction, tangential to circular path at point 'p'.

From Biot - Savart's law,

$\bar{H}$ due to infinitely long Conductor is

$$\bar{H} = \frac{I}{2\pi r} \hat{a}_\phi$$

→ In cylindrical Co-ordinate system, the $d\bar{l}$ in $\hat{a}_\phi$ direction

$$\boxed{d\bar{l} = r\, d\phi\, \hat{a}_\phi}$$

$$\oint H \cdot dl = \oint \frac{I}{2\pi \cancel{r}} \hat{a}_\phi \cdot \cancel{r} \cdot d\phi\, \hat{a}_\phi$$

$$= \int_{\phi=0}^{2\pi} \frac{I}{2\pi} \cdot d\phi$$

$$= \frac{I}{2\pi} [\phi]_0^{2\pi}$$

$$= \frac{I}{\cancel{2\pi}} [\cancel{2\pi}]$$

$$= I_{enc}$$

$$\Rightarrow \boxed{\oint H \cdot dl = I_{enc}}$$ ← Ampere's Law in integral form

Applying stoke's theorem to the L.H.S

$$\int_s (\nabla \times \bar{H})\, ds = \int_s J \cdot ds$$

$$\boxed{\nabla \times \bar{H} = J}$$ ← Ampere's Law in differential or point form

↑ Maxwell's 3rd equation.

NOTE Since, $\nabla \times \bar{H} \neq 0$; a magnetostatic field is not Conservative.

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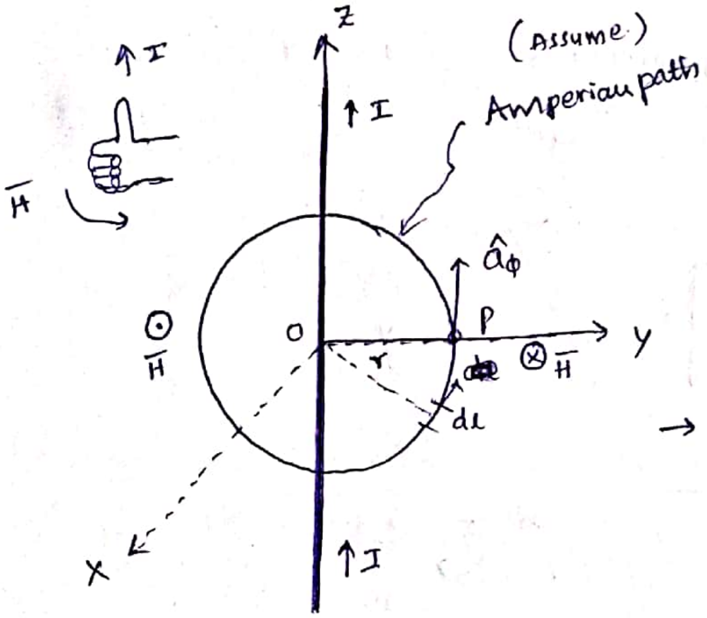
Applications of Ampere's Circuit Law :

(1) $\bar{H}$ due to Infinitely long Straight Conductor :



According to Ampere's law

$$\oint \bar{H}\cdot dl = I_{enc}$$

→ Consider closed path ≈ Amperian path which shows that $\bar{H}$ is Constant as r is Constant.

$$\bar{H} = H_r \hat{a}_r + H_\phi \hat{a}_\phi + H_z \bar{a}_z$$

$$\bar{H} = H_\phi \cdot \hat{a}_\phi \qquad [\, r \text{ is constant}, \ z=0 \text{ for point } P\,]$$

$d\bar{l}$ → elementary length ⇒ @ Cylindrical Coordinate System

$$d\bar{l} = r\cdot d\phi \, \hat{a}_\phi$$

$$\bar{H}\cdot d\bar{l} = (H_\phi \cdot \hat{a}_\phi)(r d\phi \, \hat{a}_\phi) = H_\phi \cdot r \cdot d\phi$$

According to Ampere's Circuit Law

$$\oint H\cdot dl = I_{enc}$$

$$\oint_{\phi=0}^{2\pi} H_\phi \cdot r \cdot d\phi = I \Rightarrow H_\phi \, r \cdot \int_0^{2\pi} d\phi = I$$

$$\Rightarrow H_\phi \cdot r \cdot [\phi]_0^{2\pi} = I$$

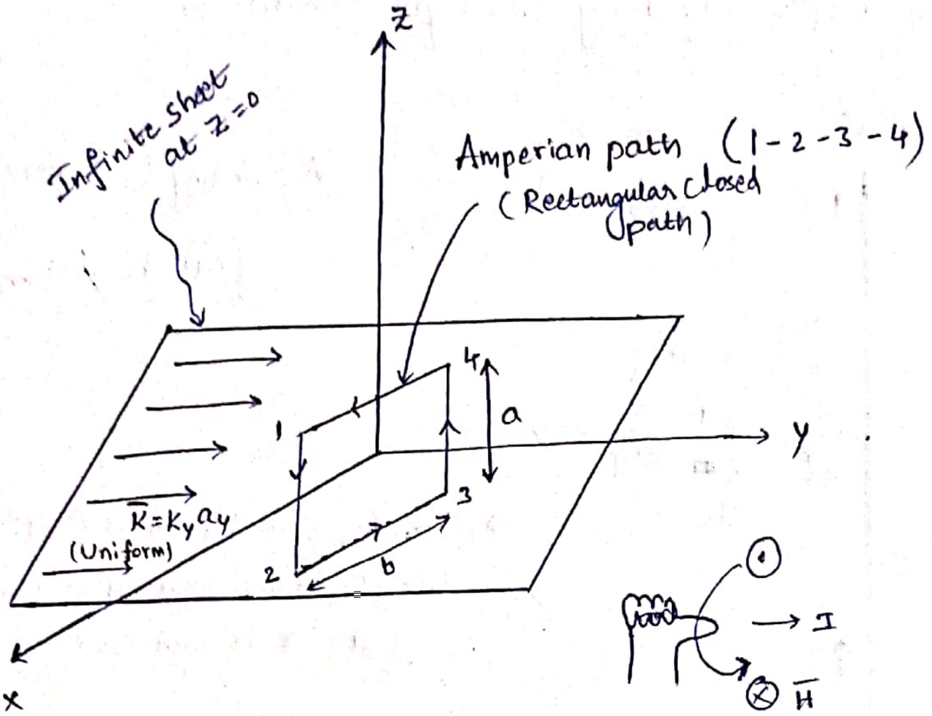
$$\Rightarrow H_\phi \cdot r \cdot [2\pi] = I$$

$$\Rightarrow H_\phi = \frac{I}{2\pi r}$$

$$\bar{H} = H_\phi \cdot \hat{a}_\phi = \frac{I}{2\pi r}\hat{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi r}\hat{a}_\phi \quad A/m}$$

(2) $\overline{H}$ due to Infinite Sheet of Current :



(a)

$K_y = \frac{I}{b} =$ Surface current density.

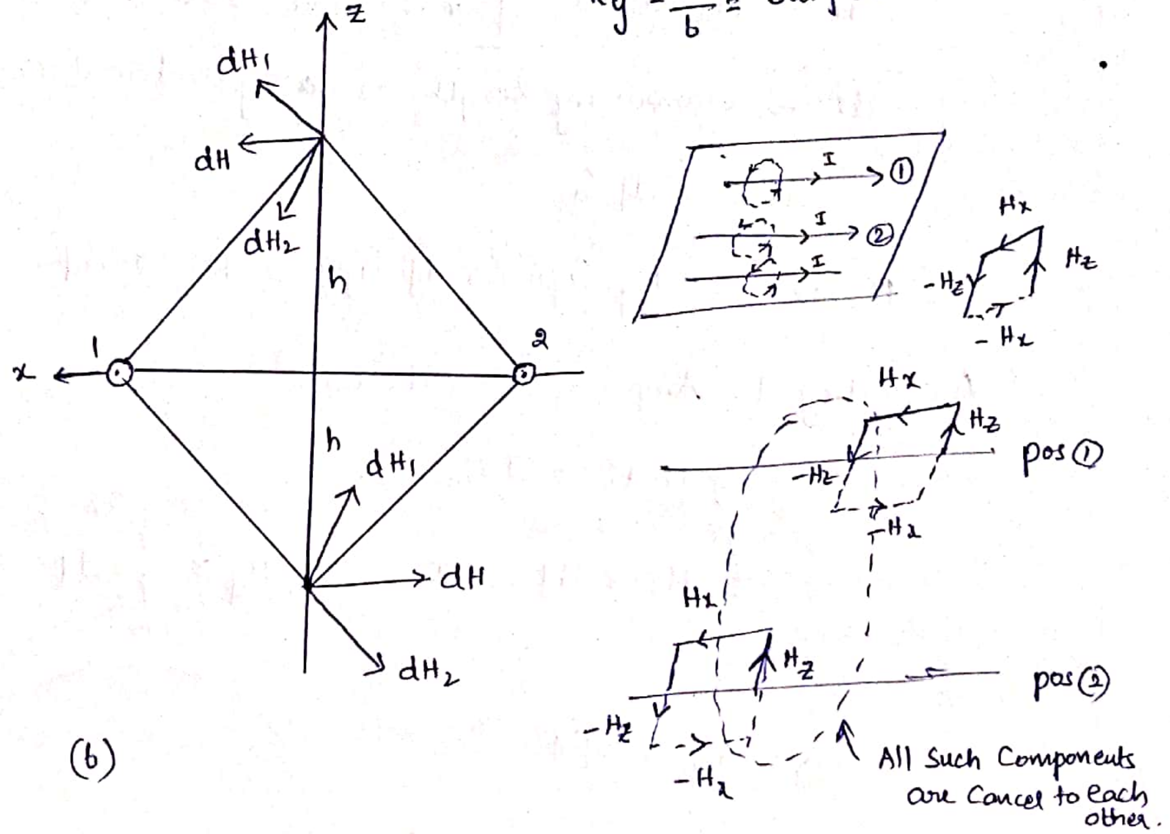


Fig: Application of Ampere's Law to an infinite sheet

(a) Closed path 1-2-3-4-1

(b) Symmetrical pair of current ~~elements~~ filaments with current along $\hat{a}_y$.

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(13)

Applying Ampere's circuit law

$$\oint H \cdot dl = I_{enc}$$

→ Consider an infinite ~~sheet~~ Current sheet in the $z=0$ plane.

→ As current is flowing in y-direction, $\bar{H}$ cannot have component in y-direction.

→ $\bar{H}$ along the z-axis is cancelled due to symmetry.

So, $\bar{H}$ has only component in x-direction.

$$\bar{H} = H_x \bar{a}_x \quad \text{for } z>0$$
$$= -H_x \bar{a}_x \quad \text{for } z<0$$

$$\boxed{\begin{aligned} \oint_L H \cdot \vec{dl} &= \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \vec{H} \cdot \vec{dl} \\ &= -\cancel{H_z}(-a) + (-H_x)(-b) + (\cancel{H_z})(a) + H_x(b) \qquad [H_z = -H_z] \\ &= 2H_x b \end{aligned}}$$

For path 1-2, $d\bar{l} = dz(-\bar{a}_z)$

For path 3-4, $d\bar{l} = dz\,\bar{a}_z$

$$\int_1^2 H \cdot dl + \int_3^4 H \cdot dl = \int_1^2 -H_x \cdot \bar{a}_x \cdot dz(-\bar{a}_z) + \int_3^4 H_x \cdot \bar{a}_x \cdot dz\,\bar{a}_z$$

$$= 0 + 0 \qquad [\because \bar{a}_x \cdot \bar{a}_z = 0]$$

$$= 0$$

For path 2-3

$$\int_2^3 H \cdot dl = \int_2^3 -H_x \cdot \bar{a}_x \cdot dx \cdot (-\bar{a}_x)$$

$$= H_x \cdot \int_2^3 dx = b \cdot H_x$$

For path 4-1

$$\int_4^1 H \cdot dl = \int_4^1 H_x \cdot \bar{a}_x \cdot dx \cdot \bar{a}_x$$

$$= H_x \int_4^1 dx = bH_x.$$

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$$\oint H\cdot dl = \int_1^2 H\cdot dl + \int_2^3 H\cdot dl + \int_3^4 H\cdot dl + \int_4^1 H\cdot dl$$

$$= 0 + b\cdot H_x + 0 + b\,H_x$$

$$= 2bH_x .$$

→ The current flowing across the distance 'b' is $K_y b$

$$I_{enc} = K_y b$$

→ According to Ampere's law

$$\oint H\cdot dl = I_{enc}$$

$$2\cancel{b}H_x = K_y\cancel{b}$$

$$\Rightarrow \boxed{H_x = \frac{1}{2}K_y}$$

Then, $\bar{H} = \frac{1}{2}K_y\,\bar{a}_x$ for $z>0$

$= -\frac{1}{2}K_y\,\bar{a}_x$ for $z<0$

In general, for an infinite sheet of current density K A/m,

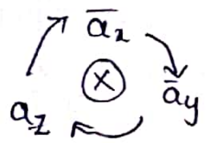
$$\boxed{H = \frac{1}{2}\bar{K}\times\bar{a}_n}$$

Here $\bar{a}_n = \bar{a}_z$

$\bar{K} = K_y\cdot\bar{a}_y$

$\bar{K}\times\bar{a}_n = K_y(\bar{a}_y\times\bar{a}_z) = K_y\cdot\bar{a}_x$

where $\bar{a}_n$ is a unit normal vector directed from the current sheet to the point of interest.



(3) $\bar{H}$ due to Infinitely Long Co-axial Cable

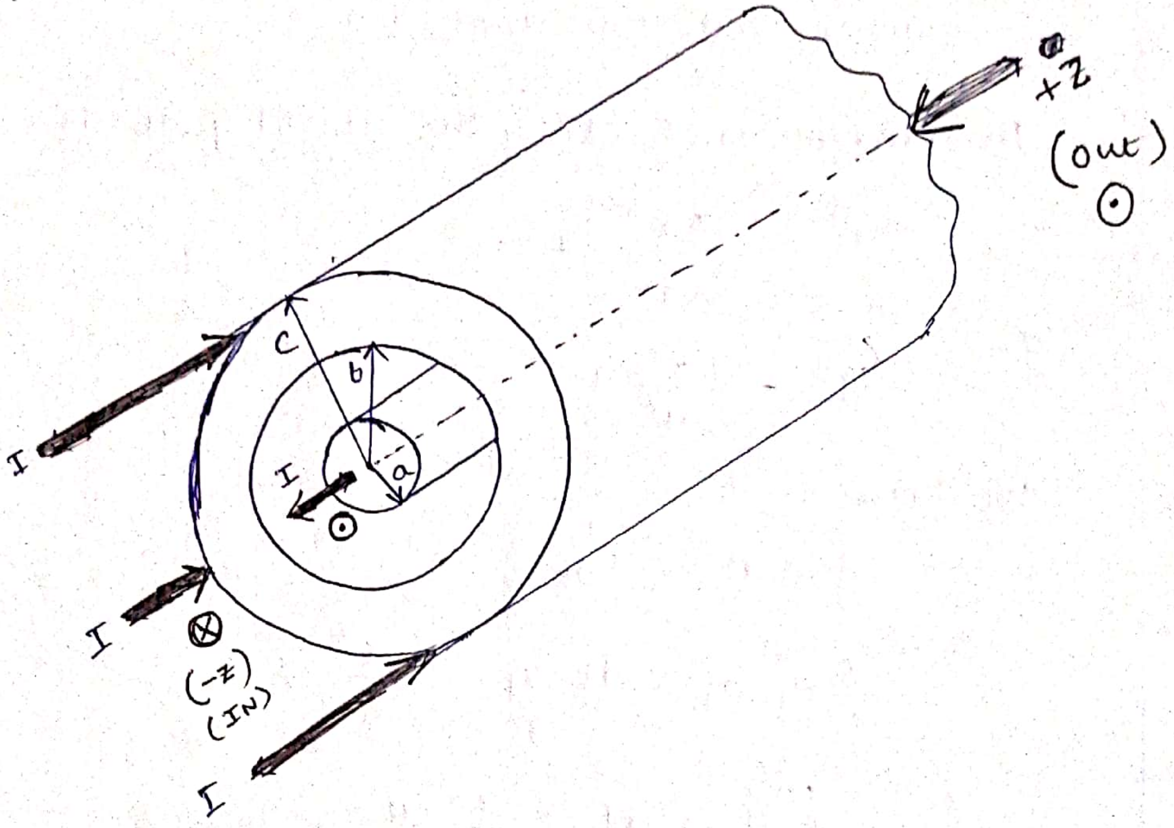
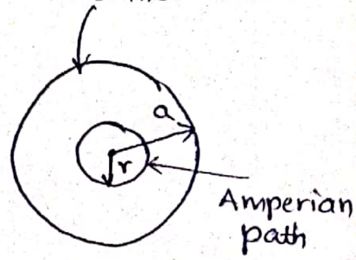


Fig: ~~Consider~~ a Co-axial Cable (Two Concentric cylinders)

→ Apply Ampere's circuit law along Amperian path for each possibility.

Region (1): With in the inner Conductor, $r < a$ $(0 \le r \le a)$

→ With in the inner Conductor, Consider an Amperian path having radius $r < a$



According to Ampere's Circuital Law

$$\oint H \cdot dl = I'$$

where I' is the Current enclosed by the closed path.

$$\bar{H} = H_\phi \cdot \bar{a}_\phi$$

$$dl = r \cdot d\phi \cdot \bar{a}_\phi$$

Area of the enclosed cross section is πr^2

→ The current, I which is flowing through inner conductor area, πa^2.

The current enclosed by the closed path is

$$I' = \frac{\cancel{\pi} r^2}{\cancel{\pi} a^2} I$$

$$I' = \frac{r^2}{a^2} I$$

We know

$$\oint H \cdot dl = I'_{enc}$$

$$\oint H_\phi \cdot \bar{a}_\phi \cdot r \cdot d\phi \cdot \bar{a}_\phi = \frac{r^2}{a^2} I$$

$$\int_{\phi=0}^{2\pi} H_\phi \cdot r \cdot d\phi = \frac{r^2}{a^2} I$$

$$H_\phi \cdot \cancel{r} \cdot [\phi]_0^{2\pi} = \frac{r \cancel{r}}{a^2} I$$

$$H_\phi \cdot (2\pi) = \frac{r}{a^2} I$$

$$H_\phi = \frac{Ir}{2\pi a^2}$$

$$\boxed{\bar{H} = \bar{H}_\phi \cdot \bar{a}_\phi = \frac{Ir}{2\pi a^2} \bar{a}_\phi} \quad A/m$$

<u>Region (2):</u> With in $a \leq r \leq b$

$$\oint H \cdot dl = I$$

$$\int_{\phi=0}^{2\pi} H_\phi \cdot a_\phi \cdot r \cdot d\phi \; \bar{a}_\phi = I$$

$$H_\phi \cdot r \cdot [\phi]_0^{2\pi} = I$$

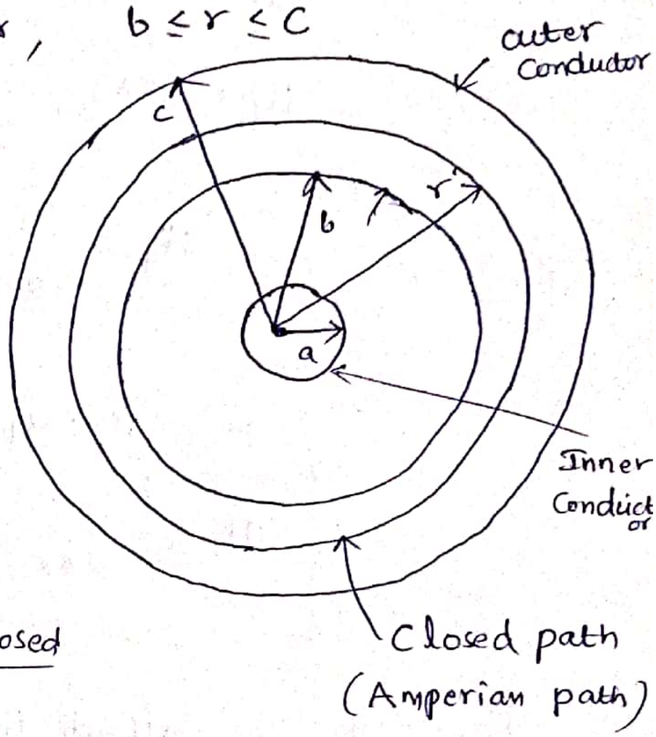
$$\Rightarrow H_\phi \cdot r \cdot (2\pi) = I$$

$$\Rightarrow H_\phi = \frac{I}{2\pi r}$$

$$\bar{H} = H_\phi \cdot \bar{a}_\phi = \frac{I}{2\pi r} \bar{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi r} \bar{a}_\phi} \quad A/m$$

Region(3): With in Outer Conductor, $b \le r \le c$



Current in outer Conductor enclosed

$$I' = \frac{\cancel{\pi}(r^2 - b^2)}{\cancel{\pi}(c^2 - b^2)} (-I)$$

$$I' = -\frac{(r^2 - b^2)}{(c^2 - b^2)} I$$

Current in inner Conductor enclosed

$$I'' = I$$

Total Current enclosed by the closed path is

$$I_{enc} = I' + I''$$

$$= -\frac{(r^2 - b^2)}{(c^2 - b^2)} I + I$$

$$= I \left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)} \right] = I \left[\frac{c^2 - \cancel{b^2} - r^2 + \cancel{b^2}}{c^2 - b^2} \right]$$

$$I_{enc} = I \left[\frac{c^2 - r^2}{c^2 - b^2} \right]$$

Apply Ampere's Law

$$\oint H \cdot dl = I_{enc}$$

$$\oint H \cdot dl = \int_{\phi=0}^{2\pi} (H_\phi \cdot \bar{a}_\phi)(r \cdot d\phi \cdot \bar{a}_\phi) = H_\phi \cdot r \, [\phi]_0^{2\pi}$$

$$= H_\phi \cdot r \, (2\pi)$$

$$\Rightarrow \quad \oint H \cdot dl = I_{enc}$$

$$H_\phi \cdot r \cdot (2\pi) = I \left[\frac{c^2 - r^2}{c^2 - b^2} \right]$$

$$H_\phi = \frac{I}{2\pi r} \left[\frac{c^2 - r^2}{c^2 - b^2} \right]$$

$$\bar{H} = H_\phi \cdot \bar{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi r} \left[\frac{c^2 - r^2}{c^2 - b^2} \right] \bar{a}_\phi} \quad A/m.$$

Region (4): Outside the Cable, $r > c$

→ Consider the closed path having radius $r > c$, Such that it closes both the Conductors.

so, $I_{enc} = +I - I = 0$

Apply Ampere's Law

$$\oint H \cdot dl = I_{enc} = 0$$

$$\boxed{\bar{H} = 0} \quad A/m$$

$$
\bar{H} = \begin{cases} \dfrac{Ir}{2\pi a^2}\,\bar{a}_\phi & \text{for } 0 \le r \le a \\[2ex] \dfrac{I}{2\pi r}\,\bar{a}_\phi & \text{for } a \le r \le b \\[2ex] \dfrac{I}{2\pi r}\left[\dfrac{c^2 - r^2}{c^2 - b^2}\right]\bar{a}_\phi & \text{for } b \le r \le c \\[2ex] 0 & \text{for } r > c \end{cases}
$$

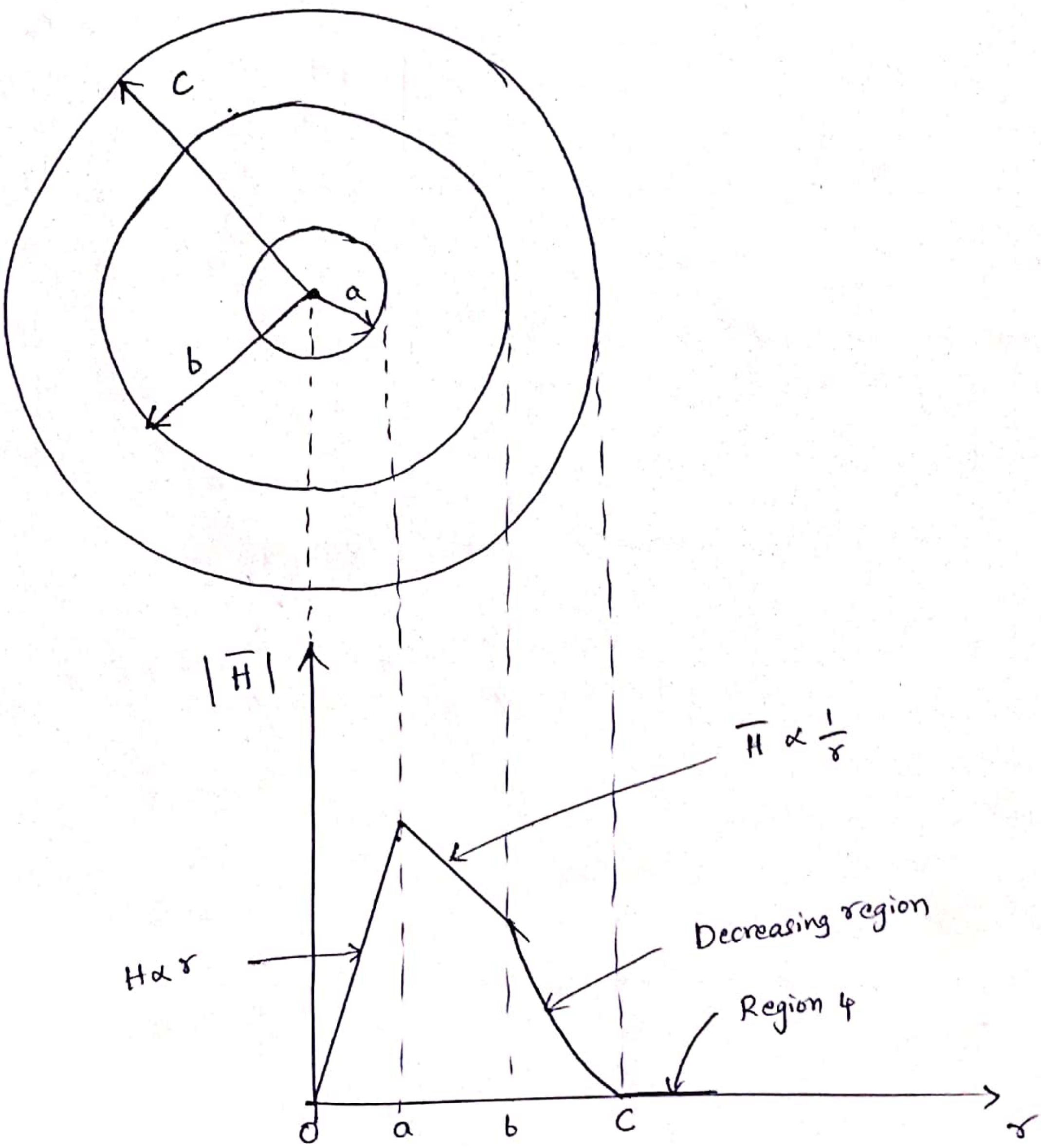


Fig: plot of $\bar{H}$ against r

MAGNETIC FLUX DENSITY - MAXWELL'S EQUATION

The magnetic flux density $\bar{B}$ is similar to the electric flux density $\bar{D}$. As $\bar{D} = \varepsilon_0 \bar{E}$ in free space, the magnetic ~~field intensity H~~ flux density $\bar{B}$ is related to the magnetic field intensity $\bar{H}$ according to

$$\boxed{\bar{B} = \mu_0 H}$$

where μ = permeability of medium

$= \mu_0 \mu_r$.

μ_0 = permeability of free space

$= 4\pi \times 10^{-7}$ H/m

μ_r = Relative permeability

~~the~~ For free space, the $\boxed{\bar{B} = \mu_0 \bar{H}}$ $[\because \mu_r = 1]$

→ The magnetic flux through a surface S is given by

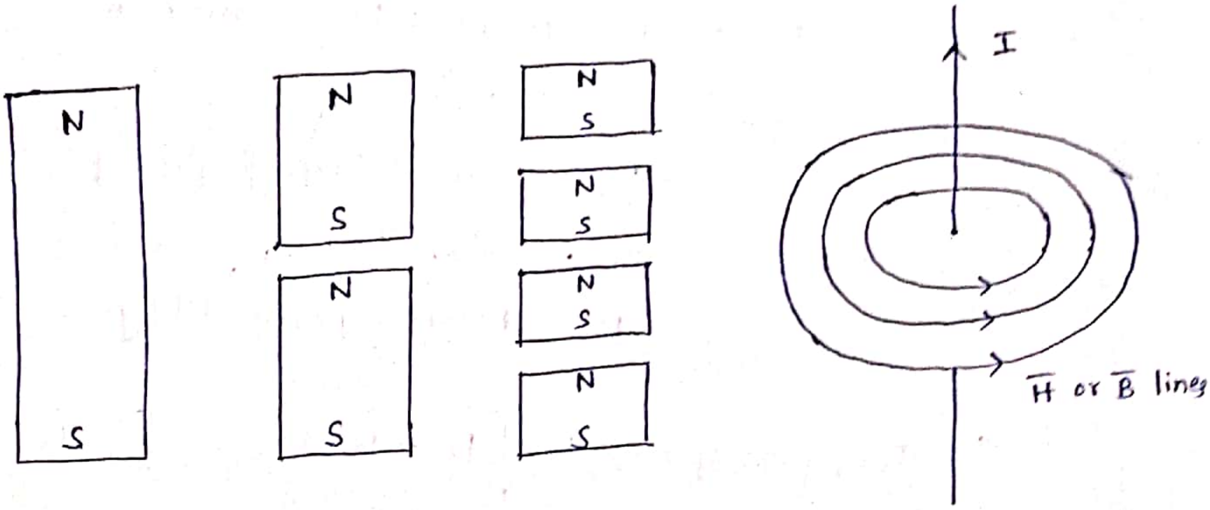
$$\boxed{\phi = \int_S B \cdot ds} \quad \text{wb}$$

→ Magnetic flux density is measured in terms of wb/m^2.

→ In case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface.

In case of magnetic field, isolated magnetic charge (i.e., pole) does not exist. Magnetic poles always occur in pair (as N-S).

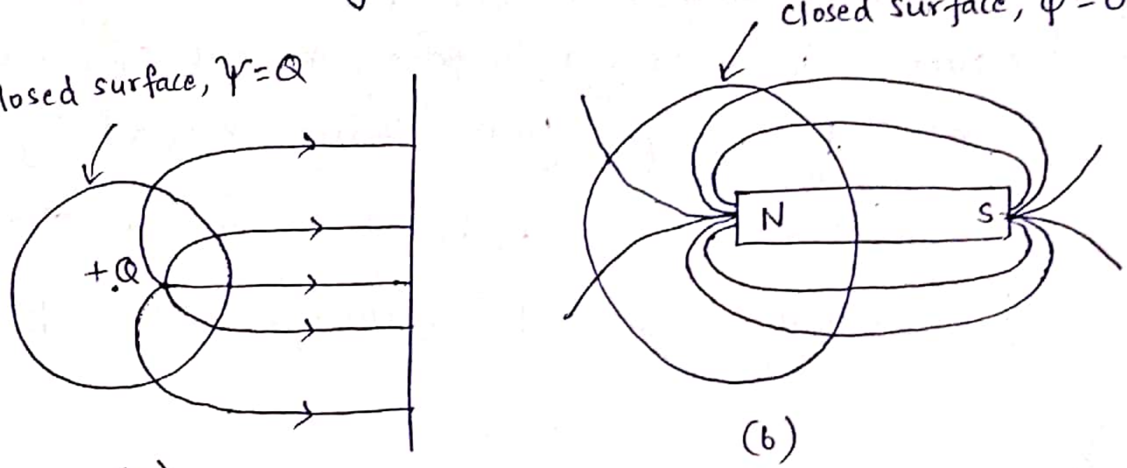
→ For example, if we desire to have an isolated magnetic pole by dividing a magnetic bar successively into two, we end up with pieces each having north (N) and South (S) poles. This process could be continued ~~till~~ until the magnets are of atomic dimensions; still we will have N-S pair occuring together. This means that ~~the~~ it is impossible to seperate the north pole from the South pole.



(a) (b)

Fig: (a) Subdivision of a magnet
(b) Magnetic field / flux lines of a straight current carrying conductor.

$\boxed{\text{NOTE}}$ An isolated magnetic pole does not exist.



(a) (b)

Fig(a): Flux leaving a closed surface due to (a) isolate electric charge $\Psi = \oint_s D \cdot ds = Q_{enc}$ (b) Magnetic charge, $\phi = \oint_s B \cdot ds = 0$

Thus, the total flux through a closed surface in a magnetic field must be zero; i.e

$$\boxed{\phi = \oint_s B \cdot ds = 0}$$

This equation is referred to as the law of conservation of magnetic flux or Gauss's law for magnetic fields, just as $\oint D \cdot ds = Q$ is Gauss's law for electrostatic fields.

→ Although the magnetostatic field is not conservative, magnetic flux is conserved.

By applying the divergence theorem to the above equation

$$\oint_s B \cdot ds = \int_v \nabla \cdot B \cdot dv = 0$$

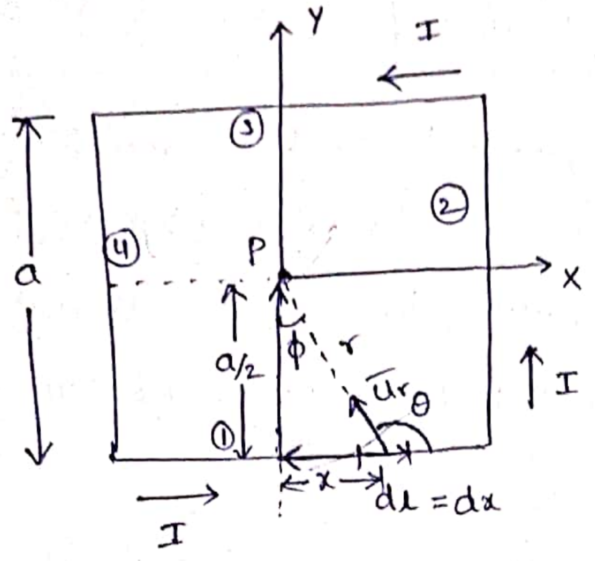
$$\boxed{\nabla \cdot \bar{B} = 0} \leftarrow \text{Maxwell's fourth Equation}$$

→ The above equations shows that magnetostatic fields have no sources or sinks.

MAXWELL'S EQUATIONS FOR STATIC FIELDS (static electric & magnetic fields)

Differential (or) point form	Integral Form	Remarks
$\nabla \cdot \bar{D} = \rho_v$	$\oint_s \bar{D} \cdot ds = \int_v \rho_v \cdot dv$	Gauss's law
$\nabla \cdot \bar{B} = 0$	$\oint_s \bar{B} \cdot ds = 0$	Non existence of magnetic mono pole
$\nabla \times \bar{E} = 0$	$\oint_L \bar{E} \cdot dl = 0$	Conservative nature of electrostatic field
$\nabla \times \bar{H} = \bar{J}$	$\oint_L H \cdot dl = \int_s J \cdot ds$	Ampere's Law

Magnetic field $\bar{B}$ at the Centre of Square loop



By using Biot-Savart's Law

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left[\frac{dl \times \bar{u}_r}{|r|^2}\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{dl \times \bar{r}}{|r|^3}\right] \qquad \left[\because \bar{u}_r = \frac{\bar{r}}{|r|}\right]$$

The total field,

$$\bar{B}_{Total} = \bar{B}_{1} + \bar{B}_{2} + \bar{B}_{3} + \bar{B}_{4}$$

$$\bar{B}_{Total} = 4\,\bar{B}_{1}$$

From the diagram,

$$\boxed{d\bar{l} = dx\,\hat{i}}$$

$$\sin\phi = \frac{x}{|r|}, \quad \cos\phi = \frac{a/2}{|r|}$$

$$\bar{r} = x(-\hat{i}) + \frac{a}{2}\hat{j}$$

$$\boxed{\bar{r} = |r|\sin\phi\,(-\hat{i}) + |r|\cos\phi \cdot \hat{j}}$$

$$\bar{r} = -|r|\sin\phi\,\hat{i} + |r|\cos\phi \cdot \hat{j}$$

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left[\frac{d\bar{l} \times \bar{r}}{|r|^3}\right]$$

$$d\bar{l} \times \bar{r} = dx\,\hat{j} \times \left[-|r|\sin\phi\,\hat{i} + |r|\cos\phi\,\hat{j}\right]$$

$$= \boxed{dx\,|r|\cos\phi}\,\hat{k} \qquad \left[\because \hat{i}\times\hat{i}=0,\ \hat{j}\times\hat{j}=0,\ \hat{i}\times\hat{j}=\hat{k}\right]$$

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left[\frac{dx\,\cancel{|r|}\cos\phi}{|r|^{\cancel{3}\,2}}\right]\hat{k}$$

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left[\frac{dx\cos\phi}{|r|^2}\right]\hat{k}$$

From the diagram

$$\tan\phi = \frac{x}{(a/2)} \qquad |r| = \frac{a/2}{\cos\phi}$$

$$\Rightarrow x = \frac{a}{2}\tan\phi$$

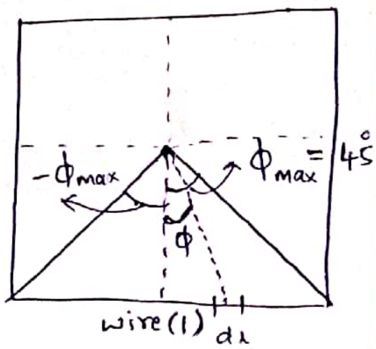
$$\Rightarrow dx = \left(\frac{a}{2}\right)\cdot\sec^2\phi\cdot d\phi$$

$$\Rightarrow dx = \left(\frac{a}{2}\right)\frac{1}{\cos^2\phi}\cdot d\phi$$

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left[\left(\cancel{\frac{a}{2}}\right)\frac{1}{\cancel{\cos^2\phi}}\cdot d\phi\cdot\frac{\cos\phi}{\frac{(a/2)^{\cancel{2}}}{\cancel{\cos^2\phi}}}\right]\hat{k}$$

$$d\bar{B} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{(a/2)}\right)\cos\phi\cdot d\phi\cdot\hat{k}$$

$$\bar{B}_1 = \int d\bar{B}_1 = \frac{\mu_0 I}{4\pi(a/2)}\int_{-\phi_{max}}^{+\phi_{max}}\cos\phi\cdot d\phi$$



$$\bar{B}_1 = \frac{\mu_0 I}{4\pi(a/2)} \left[\sin\phi\right]_{-\phi_{max}}^{\phi_{max}} \hat{k}$$

$$= \frac{\mu_0 I}{4\pi(a/2)} \left[\sin\phi_{max} - \sin(-\phi_{max})\right] \hat{k}$$

$$= \frac{\mu_0 I}{4\pi(a/2)} \left[2\sin\phi_{max}\right] \hat{k}$$

$$= \frac{\mu_0 I}{4\pi(a/2)} \times \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \hat{k} \qquad \left[\because \sin 45^\circ = \frac{\sqrt{2}}{2}\right]$$

$$\bar{B}_1 = \frac{\mu_0 I}{4\pi(a/2)} \sqrt{2} \hat{k}$$

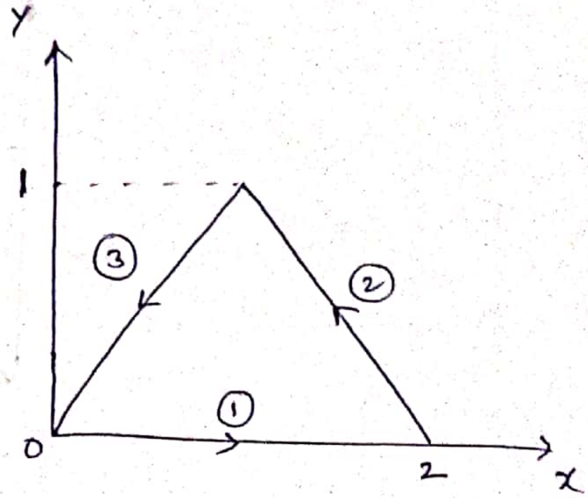
$$\bar{B}_{Total} = 4 \cdot \bar{B}_1$$

$$= \frac{\cancel{4} \mu_0 I}{\cancel{4}\pi(a/2)} \sqrt{2} \hat{k}$$

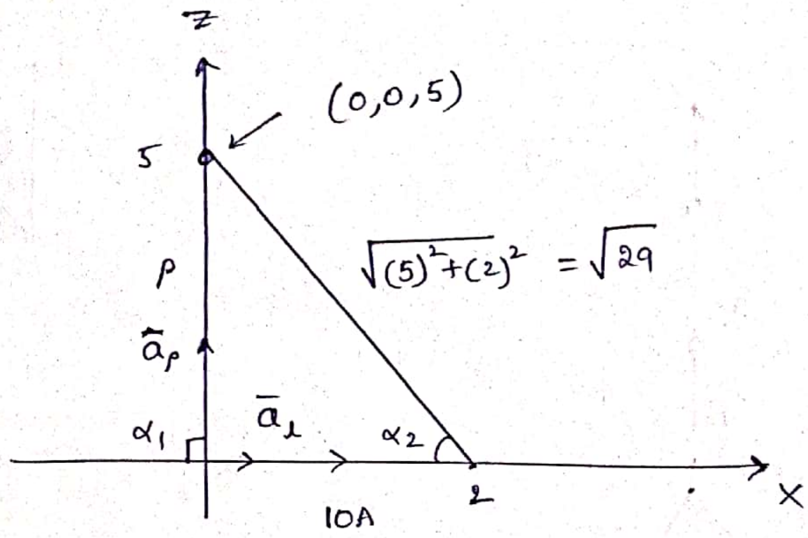
$$\boxed{\bar{B}_{Total} = \frac{\sqrt{2}}{\pi} \frac{\mu_0 I}{(a/2)} \hat{k}}$$

→ This is the field produced at the center of square loop.

Ⓟ The Conducting triangular loop shown in figure Carries a Current 10A. Find $\bar{H}$ at $(0,0,5)$ due to Side 1 of the loop.



Sol:

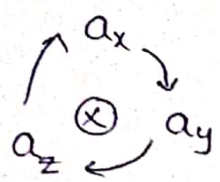


From fig: $\alpha_1 = 90^\circ$ $\qquad \cos\alpha_1 = \cos 90^\circ = 0$

$\alpha_2 = ?$ $\qquad \cos\alpha_2 = \dfrac{2}{\sqrt{29}}$; $\rho = 5$

To find $\bar{a}_\phi$: $\quad \bar{a}_l \times \bar{a}_\rho = \bar{a}_\phi$

$\bar{a}_x \times \bar{a}_z = -\bar{a}_y$

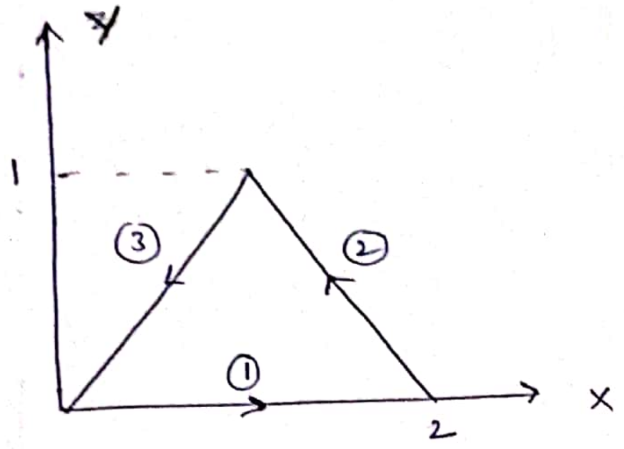


Hence, $\bar{H} = \dfrac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\bar{a}_\phi$

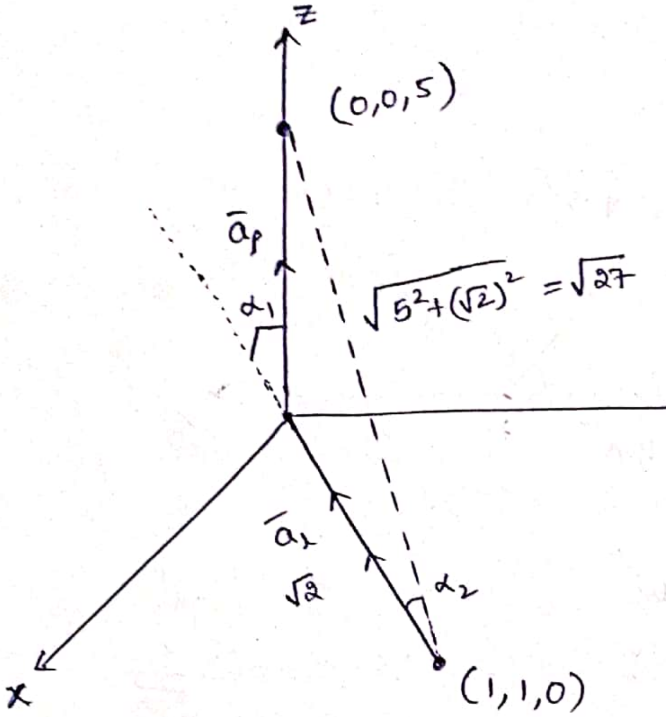
$= \dfrac{10}{4\pi(5)}\left(\dfrac{2}{\sqrt{29}} - 0\right)(-\bar{a}_y)$

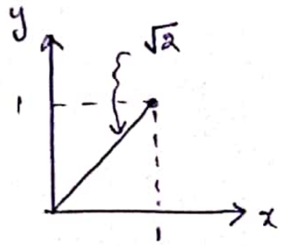
$= 0.0591\,(-\bar{a}_y) = -59.1\,\bar{a}_y$ mA/m

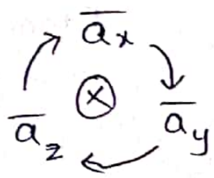
(P) Find $\overline{H}$ at (0,0,5) due to Side 3 of the triangular loop Carries Current 10A Shown in figure



Sol:







$$\cos\alpha_1 = \cos 90^\circ = 0$$

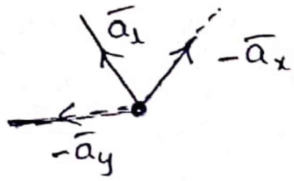
$$\cos\alpha_2 = \frac{\sqrt{2}}{\sqrt{27}} = \sqrt{\frac{2}{27}}$$

To determine: $\quad \bar{a}_\phi = \bar{a}_l \times \bar{a}_\rho$

$$= \left(\frac{-\bar{a}_x - \bar{a}_y}{\sqrt{2}}\right) \times \bar{a}_z$$

$$= \frac{-1}{\sqrt{2}}\left\{\bar{a}_x \times \bar{a}_z + \bar{a}_y \times \bar{a}_z\right\}$$

$$= \frac{-1}{\sqrt{2}}\left[-\bar{a}_y + \bar{a}_x\right]$$



$$\bar{l} = -\bar{a}_x - \bar{a}_y$$

$$\bar{a}_l = \frac{-\bar{a}_x - \bar{a}_y}{\sqrt{2}}$$

$$\bar{a}_\rho = \bar{a}_z$$

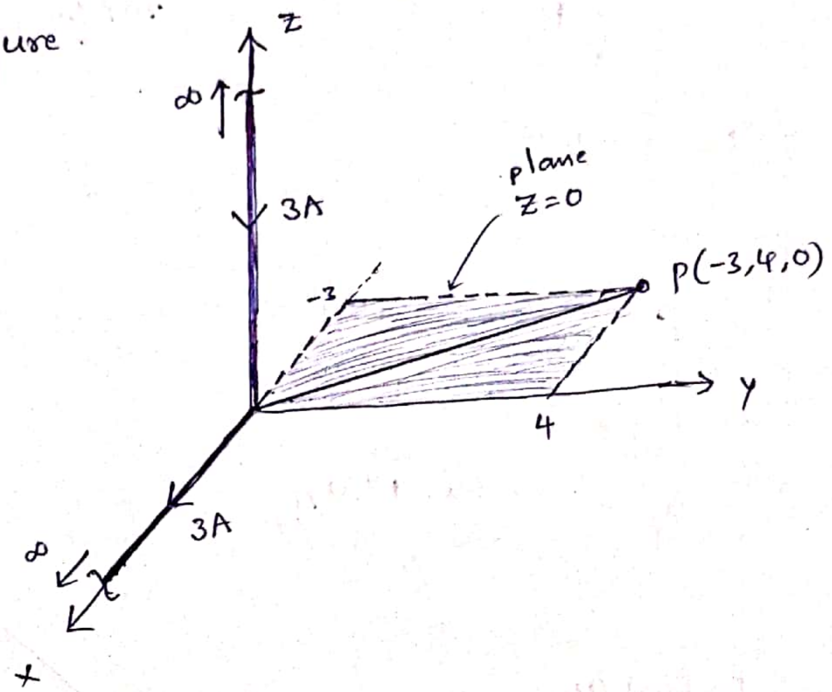
$$\therefore \; H_3 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\bar{a}_\phi$$

$$= \frac{10}{4\pi(5)}\left(\frac{\sqrt{2}}{\sqrt{27}} - 0\right)\left(\frac{-\bar{a}_x + \bar{a}_y}{\sqrt{2}}\right)$$

$$= 0.03063\,(-\bar{a}_x + \bar{a}_y)$$

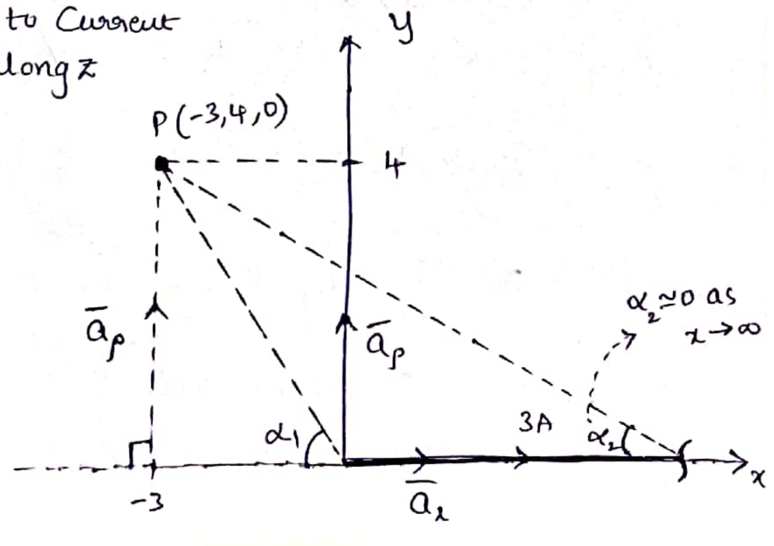
$$\boxed{H_3 = -30.63\,\bar{a}_x + 30.63\,\bar{a}_y \;\; mA/m}$$

(P) Find $\bar{H}$ at $(-3,4,0)$ due to Current filament shown in figure.



Sol: Let $\bar{H} = \bar{H}_1 + \bar{H}_2$ .... at point $P(-3,4,0)$

Due to Current along x — Due to Current along z

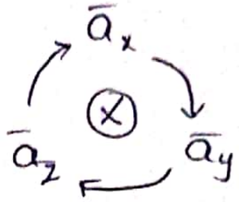


$$\cos\alpha_1 = \frac{3}{\sqrt{4^2+3^2}} = \frac{3}{\sqrt{25}} = \frac{3}{5}$$

$$\cos\alpha_2 = \cos(0) = 1$$

To find, $\bar{a}_\phi = \bar{a}_l \times \bar{a}_\rho$

$= \bar{a}_x \times \bar{a}_y = \bar{a}_z$



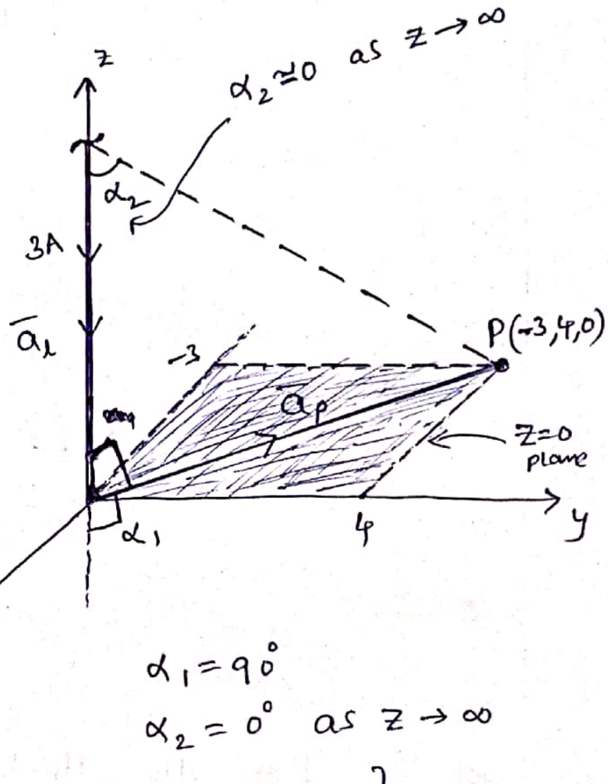
To find $\bar{H}_1$:

$$\bar{H}_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\bar{a}_\phi$$

$$= \frac{3}{4\pi(4)}\left(1 - \frac{3}{5}\right)\bar{a}_\phi$$

$$\boxed{\bar{H}_1 = 23.88\,\bar{a}_z \text{ mA/m}}$$

To find $\bar{H}_2$:



$\bar{\rho} = -3\bar{a}_x + 4\bar{a}_y$

$\rho = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$$\bar{a}_\rho = \frac{-3\bar{a}_x + 4\bar{a}_y}{5}$$

$\alpha_1 = 90^\circ$

$\alpha_2 = 0^\circ$ as $z \to \infty$

$\left\{ \begin{array}{l} \cos\alpha_1 = 0 \\ \cos\alpha_2 = 1 \end{array} \right\}$

To find $\bar{a}_\phi$

$\bar{a}_\phi = \bar{a}_l \times \bar{a}_\rho$

$$= -\bar{a}_z \times \left(\frac{-3\bar{a}_x + 4\bar{a}_y}{5}\right)$$

$$= -\bar{a}_z \times \left(-\frac{3}{5}\bar{a}_x + \frac{4}{5}\bar{a}_y\right)$$

$$= \frac{3}{5}\bar{a}_y + \frac{4}{5}\bar{a}_x$$

$$\bar{a}_\phi = \left(\frac{4\bar{a}_x + 3\bar{a}_y}{5}\right)$$

$$\overline{H}_2 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\overline{a_\phi}$$

$$= \frac{3}{4\pi(5)}(1-0)\cdot\overline{a_\phi}$$

$$= \frac{3}{4\pi(5)} \times \left(\frac{4\overline{a}_x + 3\overline{a}_y}{5}\right)$$

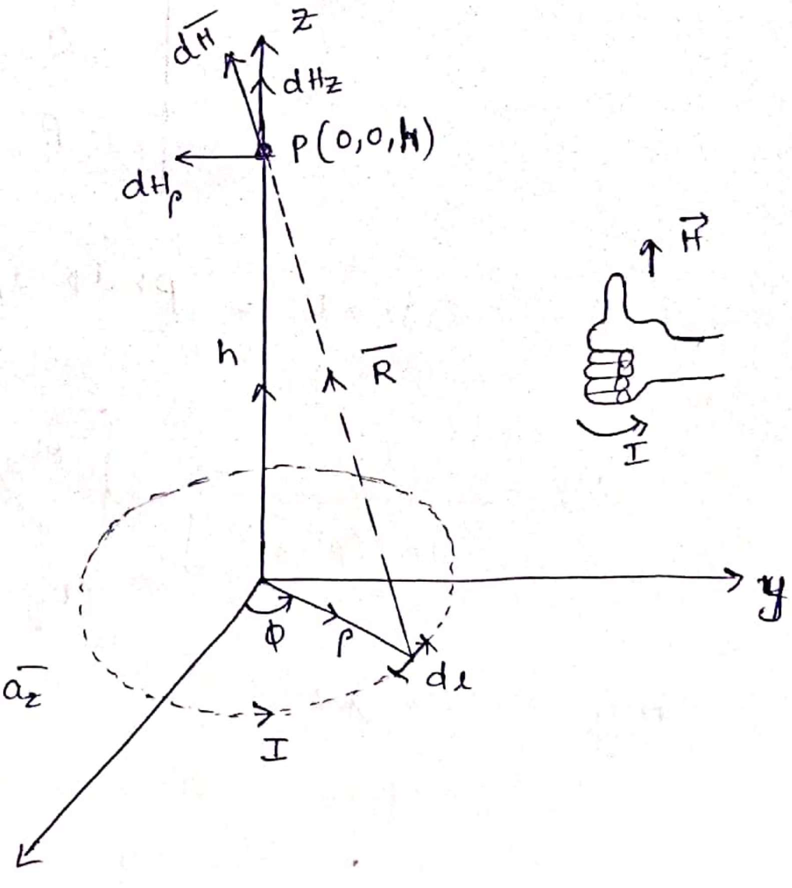
$$\boxed{H_2 = 38.2\,\overline{a}_x + 28.65\,\overline{a}_y \ mA/m}$$

So

$$\overline{H} = \overline{H}_1 + \overline{H}_2$$

$$\overline{H} = 38.2\,\vec{a}_x + 28.65\,\overline{a}_y + 23.88\,\overline{a}_z \ mA/m$$

(P) A circular loop located on $x^2+y^2=9$, $z=0$ carries a direct current of 10A along $\overline{a}_\phi$. Determine $\overline{H}$ at $(0,0,4)$ and $(0,0,-4)$.



Sol:

At point $(0,0,h)$

Using Biot-Savart's law

$$d\overline{H} = \frac{I\,d\ell \times \overline{R}}{4\pi R^3}$$

Cylindrical Co-ordinate System

$$d\overline{l} = d\rho\,\overline{a}_\rho + \rho\,d\phi\,\overline{a}_\phi + dz\,\overline{a}_z$$

Since ρ = Constant

& $z=0$ for given loop

so, $d\rho = dz = 0$

$$\boxed{d\overline{l} = \rho\,d\phi\cdot\overline{a}_\phi}$$

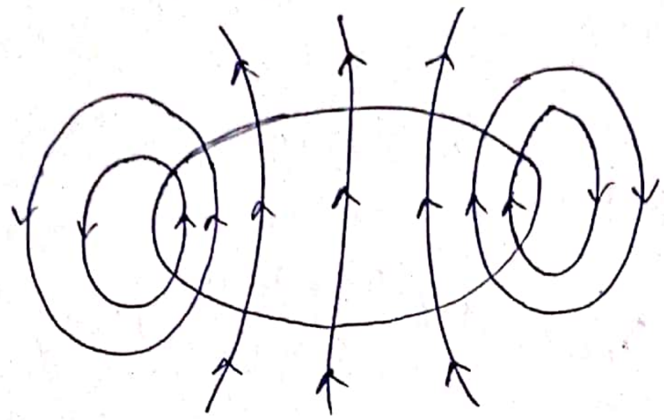


Fig: Flux lines due to the Current loop.

From Fig:

$$\rho \bar{a}_\rho + \bar{R} = h\bar{a}_z$$

$$\bar{R} = h\bar{a}_z - \rho\bar{a}_\rho$$

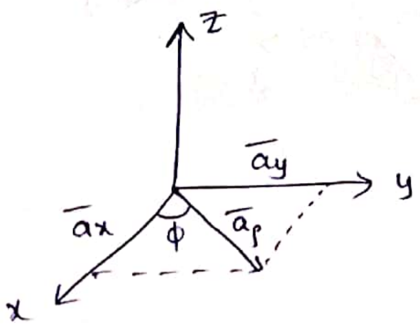
$$d\bar{l} \times \bar{R} = \begin{vmatrix} \bar{a}_\rho & \bar{a}_\phi & \bar{a}_z \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix}$$

$$d\bar{l} \times \bar{R} = \rho h d\phi \, \bar{a}_\rho + \rho^2 d\phi \cdot \bar{a}_z$$

$$d\bar{H} = \frac{I}{4\pi\left[(\rho^2+h^2)^{3/2}\right]} \left\{ \cancelto{0}{\rho h d\phi \, \bar{a}_\rho} + \rho^2 d\phi \cdot \bar{a}_z \right\}$$

(By symmetry)

NOTE:



$$\bar{a}_\rho = \cos\phi \, a_x + \sin\phi \, \bar{a}_y$$

Integrating Cosϕ & Sinϕ over $0 \le \phi < 2\pi$ gives Zero.

$$\bar{H} = \int d\bar{H}\ \bar{a}_z = \int \frac{I\rho^2\, d\phi\ \bar{a}_z}{4\pi\left[\rho^2+h^2\right]^{3/2}}$$

$$= \frac{I\rho^2\,\bar{a}_z}{4\pi\left[\rho^2+h^2\right]^{3/2}} \int_0^{2\pi} d\phi$$

$$= \frac{I\rho^2\,\bar{a}_z}{{}^{2}\cancel{4}\pi\left[\rho^2+h^2\right]^{3/2}}\ \cancel{2}\pi$$

$$\boxed{\bar{H} = \frac{I\rho^2\,\bar{a}_z}{2\left[\rho^2+h^2\right]^{3/2}}}$$

---> MFI at point (0,0,h) due to circular current loop (at $z=0$) having radius ρ.

To find ρ: $x^2+y^2=\rho^2$

$x^2+y^2=(3)^2$ ... given

so, $\rho=3$.

(a) At (0,0,4)

put $I=10A$, $h=4$

$$\bar{H}(0,0,4) = \frac{10(3)^2}{2\left[9+16\right]^{3/2}}\ \bar{a}_z$$

$$= 0.36\ \bar{a}_z\ A/m$$

(b) At (0,0,−4), $h=-4$, $I=10A$.

$$\bar{H} = \frac{I\rho^2}{2(\rho^2+h^2)^{3/2}}\ \bar{a}_z = \frac{10(3)^2\ \bar{a}_z}{2\left[9+(-4)^2\right]^{3/2}} = 0.36\ \bar{a}_z\ A/m$$

So $\bar{H}(0,0,4) = \bar{H}(0,0,-4) = 0.36\ \bar{a}_z\ A/m$

(P) planes $z=0$ and $z=4$ carry current $K = -10\,a_x$ A/m and $K = 10\,a_x$ A/m, respectively. Determine $\bar{H}$ at

(a) $(1,1,1)$ (b) $(0,-3,10)$

Sol: The parallel ~~sheets~~ current sheets are shown in Figure

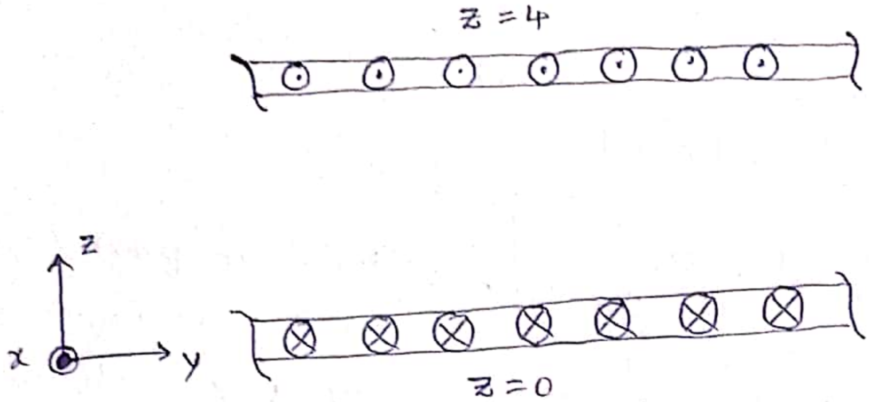


Fig: parallel infinite current sheets

Let $H = H_0 + H_4$

where H_0 and H_4 are the contributions due to current sheets $z=0$ and $z=4$ respectively.

(a) At $(1,1,1)$, which is between the plates $(0 < z=1 < 4)$

$$H_0 = \frac{1}{2} K \times a_n = \frac{1}{2}(-10\bar{a}_x) \times \bar{a}_z = 5\bar{a}_y \text{ A/m}$$

$$H_4 = \frac{1}{2} K \times a_n = \frac{1}{2}(10\,a_x) \times (-\bar{a}_z) = 5\,\bar{a}_y \text{ A/m}$$

Hence $\vec{H} = 10\,\bar{a}_y$ A/m

(b) At $(0,-3,10)$, which is above the two sheets $(z=10>4>0)$

$$H_0 = \frac{1}{2} K \times a_n = \frac{1}{2}(-10\bar{a}_x) \times \bar{a}_z = 5\bar{a}_y \text{ A/m}$$

$$H_4 = \frac{1}{2} K \times a_n = \frac{1}{2}(10\bar{a}_x) \times \bar{a}_z = -5\,\bar{a}_y \text{ A/m}$$

Hence $\vec{H} = 0$ A/m.

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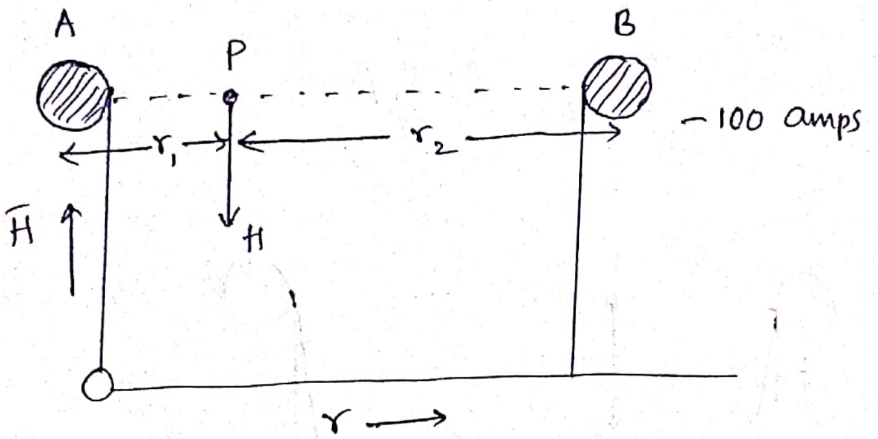
(P) A Single phase circuit Comprises two parallel Conductors A & B, 1 Cm diameter and Spaced 1m apart. The Conductor Carry Currents of +100 and −100 amps., respectively. Determine the field intensity at the surface of each Conductor and also in the space exactly midway between A & B.



Sol

Field at any point 'p' between A & B is

$$H = H_A + H_B$$

$$= \frac{I}{2\pi r_1} + \frac{I}{2\pi r_2}$$

$$= \frac{I}{2\pi}\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

H at Conductor surface

$$\bar{H} \simeq \frac{100}{2\pi}\left[\frac{1}{1\times10^{-2}\times\frac{1}{2}} + \frac{1}{1}\right]$$

$$\simeq \frac{100}{2\pi}\left[\frac{100}{0.5} + \frac{1}{1}\right]$$

$$= \frac{10050}{\pi} \text{ Amp/m}$$

$\bar{H}$ at midway between A & B

$$= \frac{I}{2\pi(0.5)} + \frac{I}{2\pi(0.5)}$$

$$= \frac{100}{\pi} \times 4$$

$$= \frac{200}{\pi} \text{ Amps/m.}$$

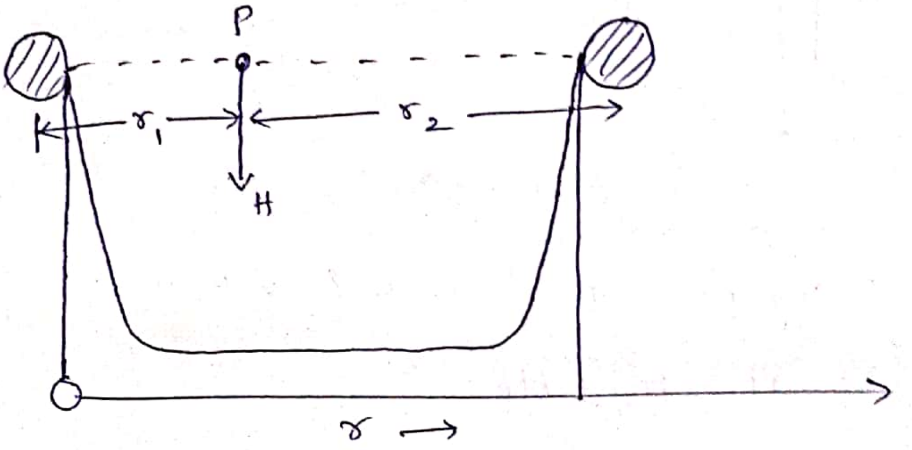


Fig: Variation of $\bar{H}$ b/w A & B.